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Mixed strategy trade equilibria

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Abstract. Two major paradigms prevail in trade theory: perfect competition and monopolistic competition. In the former case only prices are important, whereas in the latter case product characteristics are important. In both instances pure-strategy equilibria exist. This paper considers the hybrid situation where some customers care only about the price while others have a brand preference or care about product characteristics. It is shown that an equilibrium in pure strategies does not exist, but that a mixed-strategy equilibrium does exist. This in turn implies endogenous uncertainty in the pattern, terms, and the gains from trade.

Equilibres du commerce international en stratégies mixtes. Il existe deux paradigmes majeurs dans la théorie du commerce international: la concurrence parfaite et la concurrence monopolistique. Dans le premier cas, seuls les prix sont importants, alors que dans le second cas les caractéristiques des produits sont importants. Dans les deux cas, des équilibres existent en stratégies pures. Cet article examine la situation hybride où certains clients ne portent attention qu'aux prix alors que d'autres ont des préférences pour des marques de commerce ou pour certains caractéristiques des produits. On montre qu'il n'existe pas d'équilibre en pures stratégies, mais qu'il existe un équilibre en stratégies mixtes. Voilà qui entraîne de l'incertitude endogène dans le pattern du commerce international, ainsi que dans les termes d'échange et les gains qui découlent du commerce international.

I. INTRODUCTION

Two polar paradigms have been used to model international trade flows. Models of perfect competition assume firms sell homogeneous products and engage in marginal cost pricing. This class of model, for the most part, generates the classical predictions of one-way trade and specialization. In contrast, models of monopolistic

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competition view international markets as places where firms sell slightly differentiated products. These differences may arise for a number of reasons, including differences in quality or variety. The predictions of monopolistically competitive international trade models are typically very different from those of perfect competition. These models generate, for example, intra-industry trade, prices in excess of marginal cost, and the absence of complete specialization.[1]

The importance of product characteristics depends on consumer preferences. For some consumers, prices are the sole factor in the purchase decision; for others, product characteristics affect the purchase decision. The extant trade literature has essentially assumed that consumers either care about characteristics or do not care about characteristics. In this paper we focus on the hybrid situation where some consumers seek to buy at the lowest price without regard for product characteristics, whereas others have a brand or country of origin preference. Such a partition is commonly observed; see Varian (1980), Narasimhan (1988), Raju et al. (1990), and Neven et al. (1991). One example of a market where our model is applicable is the transatlantic airline market, where some consumers (within a certain price range) prefer to fly a particular airline (say American or Lufthansa), while others seek the lowest-priced ticket.

As is well known, under perfect competition and monopolistic competition there exist pure-strategy Nash equilibria. It turns out, however, that in the hybrid situation where some customers care only about price while others value characteristics, no pure-strategy equilibrium exists. The intuition is that a competitor can always exploit a pure (fixed) strategy, by slightly undercutting this price and thereby capturing all of the price conscious customers. However, a suitable randomization of the product price precludes the rival from knowing what price to charge to undercut the price. In fact, we show that a randomized-(mixed) pricing Bertrand-Nash equilibrium does exist. The interesting feature of our model is that the combination of perfect competition and monopolistic competition generates 'endogenous uncertainty,' since in equilibrium firms randomize their prices for strategic reasons. Our aim is to analyse how this 'endogenous uncertainty' affects the pattern, terms, and gains from trade. This is in sharp contrast to other models of trade, including those with perfect competition (cf. Helpman and Razin 1978) and monopolistic competition (cf. Owen and Perrakis 1986), where uncertainty stems from an exogenous source.

It is useful to contrast our model with others appearing in the trade literature. In Neven, Norman, and Thisse (1991), consumers are assumed to value identically the domestic good, but valuations of the foreign good vary according to a smooth distribution. It is this smoothness that generates a pure strategy equilibrium. In our model there are only two types of brand preferences, and mixed strategies provide the smoothing necessary to obtain equilibrium.[2] Fisher and Wilson (1987), on the

1 A third category of model focuses on quantity competition in homogeneous goods, see Brander (1981).

2 See Milgrom and Weber (1985) for an analysis of the relationship between these modeling devices.

other hand, assume that all consumers are identical, but that a price wedge always exists between the foreign and domestic market. This is isomorphic to assuming limited brand loyalty, where consumers switch if a threshold price difference is exceeded; see Raju et al. (1990) and Shilony (1977) for an analysis along these lines. Depending on whether the threshold price difference is large or small, a pure or mixed-strategy solution results.

Our paper is organized as follows. Section II describes our general model, while section III shows that monopoly and perfect competition obtain as special cases. Section IV characterizes equilibrium in the intermediate case where there are both brand-loyal and price-conscious consumers. Trade issues are examined in section V.

II. MARKET STRUCTURE

We focus on trade in similar products between two countries, with one firm located in each country. The firms are indexed by $i[i = 1, 2]$, with firm '1' being located at home and firm '2' abroad. Firms engage in price competition and attempt to maximize expected profits, $E\pi_i$, where $\pi_i = P_iQ_i - C_i(Q_i); P_i$ is firm i's product price, Q_i is the quantity sold by firm i, and C_i are firm i's costs. The cost function for firm i is given by $C_i(Q_i) = M_iQ_i$, where M_i denotes marginal cost. Thus, marginal costs may differ, owing to transportation costs, tariffs, or a technological advantage. The situation where $M_1 < M_2$ corresponds to Ricardo's view of absolute advantage, whereas $M_1 = M_2$ implies no absolute advantage.

On the demand side there are two types of customers: price-conscious customers (who do not have a brand preference) and brand-conscious customers (who have a brand preference). Price-conscious customers each purchase one unit of product from the lowest pricing firm, provided the price is at or below their reservation price, R. Otherwise, they refuse to purchase. In total there are $I \equiv I_h + I_f$ price-conscious consumers; I_h of these consumers are located in the home country and I_f are located in the foreign country.

Brand-conscious customers, on the other hand, have a preference for a particular firm's product. One may think of this loyalty as arising along nationalistic lines ('buy American' or 'buy British'), or from a preference for particular product characteristics (California vs. French wine). Customers with a brand preference will purchase one unit of their preferred firm's product, provided the price does not exceed the reservation price, R. If the price of their preferred brand is above R, they will purchase the other brand if its price is no greater than R.[3] The total number of customers with a preference for firm 1's product is $B_1 = B_{1f} + B_{1h}$, where the subscript 1 denotes a preference for firm 1's product, and the subscripts h and f indicate whether the customers with a brand preference are located in the home or foreign market. Similarly, the total number of customers with a preference

3 As a referee notes, it is possible to extend the model to allow for differences in reservation prices for the two goods.

for firm 2's product is $B_2 = B_{2f} + B_{2h}$. Note that each firm has a market at home and abroad, and markets are not segmented.

Given the above framework, the demand for firm 1's product depends on the price charged by the rival firm. We impose the following structure on the demand for firm 1's product; a similar structure applies for firm 2:

$$Q_1 = \begin{cases} B_1 + I & \text{if } P_1 < P_2 \leqq R \\ B_1 & \text{if } P_2 < P_1 \leqq R \\ B_1 + I/2 & \text{if } P_1 = P_2 \leqq R \\ 0 & \text{if } P_1 > R. \end{cases} \tag{1}$$

Note that, provided firm 1 prices at or below R, it is assured of getting the B_1 customers who have a brand preference for its product. If firm 1 charges the lowest price, it also gets the I price-conscious customers. Thus, if firm 1 charges the lowest price and it does not exceed R, it services in total, $B_1 + I$ customers. On the other hand, if the other firm charges a lower price than 1, firm 1 loses the price-conscious customers and services only its B_1 brand-loyal customers. For completeness, it is assumed that firms share equally the price-conscious customers if identical prices are charged.

The remainder of this paper considers various special cases of the above model and examines implications for trade issues. For example, the above model captures the flavour of perfect competition when all customers are price conscious ($I > 0, B_1 = B_2 = 0$). The model characterizes monopolistic competition when consumers care about product characteristics ($I = 0, B_1 > 0$, and $B_2 > 0$). The analysis also examines implications of a mixture of these two cases, that is, when some consumers have a brand preference and others simply care about price. This situation obtains when I, B_1, and B_2 all are positive.

III. POLAR CASES

1. Perfect competition

We begin our analysis with the situation where products are viewed as perfect substitutes by all consumers ($B_1 = B_2 = 0$ and $I > 0$), and where firms have identical marginal costs ($M_1 = M_2 = M < R$). Each firm chooses a price to maximize profits. When there are only price-conscious consumers and the two firms have identical marginal costs of M, the symmetric Bertrand-Nash equilibrium prices are $P_1 = P_2 = M$, and each firm services $I/2$ customers. To verify these claims, it is sufficient to show that $P_1 = P_2 = M$ satisfies the mutual best-response property. Given that firm 2 sets $P_2 = M$, firm 1 has no incentive to price above M (since it would lose all its customers), nor does it have an incentive to price below M (since profits would be negative if it priced below marginal cost). Similarly, given that firm 1 sets $P_1 = M$, firm 2 has no incentive to deviate from $P_2 = M$. Presumably, if the consumers are divided equally across countries, no trade will take place.

Note that if firm 1 enjoyed an absolute advantage over firm 2 ($M_1 < M_2 < R$), then in equilibrium firm 1 would capture the entire market.[4] While it is a bit presumptuous to discuss trade issues within a partial equilibrium model, given familiarity with Ricardo's trade model one can imagine the trade configurations that would develop if the foreign country possessed a comparative advantage in some other product. In particular, if there exists another product that the foreign country produces at a lower cost than the home country, this would imply Ricardian one-way trade with complete specialization.

2. Monopolistic competition

We next examine the polar case of monopolistic competition, which arises when all customers have a brand preference ($B_1 > 0, B_2 > 0$, and $I = 0$). All customers are brand loyal provided $P_i \leqq R$, but they will switch to buying the other product if $P_i > R$. While this is not the most general case conceivable (such as continuous substitutability between brands), it does capture the essence of models of trade in differentiated products. Assuming $M_i \leqq R$, the Bertrand-Nash equilibrium prices are $P_1 = P_2 = R$, and the corresponding quantities are $Q_1 = B_1; Q_2 = B_2$. This obtains regardless of differences in marginal costs. In order to verify these assertions, note that neither firm has an incentive to price above R (since customers would switch to the other brand and the firm would sell no product). Pricing below the reservation price does not increase demand, since the elasticity of demand is zero for prices below R. Hence, in equilibrium, each firm prices at the reservation price (the monopoly price) and fully exploits the zero-one demand structure of their brand-loyal customers.

The pattern of trade in this scenario is straightforward: country 1 exports B_{1f} and country 2 exports B_{2h}. This immediately establishes the intra-industry nature of trade; that is, there is two-way trade in similar products. Again, one can append this sector with another product sector, competitive or otherwise, and analyse trade issues. For example, one could allow for differences in reservation prices or analyse the consequences of a tariff wedge.

IV. THE INTERMEDIATE CASE

While the above models of perfect competition and monopolistic competition are highly stylized, they convey the flavour of two modern trade-theory paradigms. In this section we investigate the hybrid situation where some customers care about product characteristics, while others care only about price. We shall show that this more realistic case necessitates a solution different from the pure-strategy equilibria investigated for the two polar cases. As in Varian (1980) and Narasimhan (1988), a characterization of the Nash equilibrium requires randomized- (mixed) pricing strategies.

4 While there is a continuum of epsilon and mixed strategy Nash equilibria with this configuration of costs, in all equilibria the low-cost firm prices below the high-cost firm with probability one and thus captures the entire market.

Suppose that firms have identical marginal costs $M_1 = M_2 = M < R$. For notational convenience assume that each firm enjoys the same number, B, of brand-loyal customers ($B_1 = B_2 = M > 0$), and let there be $I > 0$ price-conscious customers. Given this simple set-up, we have the following 'negative' result.

PROPOSITION 1. *Given brand-loyal and price-conscious consumers, there does not exist a pure-strategy Bertrand-Nash trade equilibrium.*

Proof. The proof proceeds by contradicting the mutual best-response property of any price pair $(P_1, P_2) \in R_+^2$. However, we first demonstrate that the relevant strategy space is $[\hat{P}, R]$, where

$$\hat{P} \equiv M + (R - M)\frac{B}{B+I}. \tag{2}$$

Note that if a firm charges $\hat{P}$, its profits are, at most,[5] $(R-M)B$. If a firm charges a price below $\hat{P}$, its profits must thus be less than $(R-M)B$. Hence, a firm would never charge a price below $\hat{P}$ as it could always earn at least $(R-M)B$ by charging the reservation price, R. Note also that a firm would never charge a price above R, since consumers demand zero units at such prices. Hence candidates for a pure-strategy equilibrium must lie in the interval $[\hat{P}, R]$.

By way of contradiction, suppose $(P_1^*, P_2^*) \in [\hat{P}, R] \times [\hat{P}, R]$ is a Bertrand-Nash equilibrium. We consider two cases.

Case 1: $\hat{P} < P_2^* \leqq R$. Then firm 1's ϵ-best response to P_2^* is to charge ϵ small below P_2^*, since $\pi_1(P_2^* - \epsilon) > \pi_1(P_2^*)$ and $\pi_1(P_2^* - \epsilon) > (R-M)B > \pi_1(P_1)$ for any $P_1 \in [P_2^*, R)$. These inequalities follow because, if firm 1 priced at firm 2's price, it would have to share the price-conscious consumers, while if it priced above firm 2's price, it would lose the price-conscious customers and service only B brand-loyal customers. Given that firm 1 charges $P_1^* \equiv P_2^* - \epsilon$, it pays firm 2 to deviate from P_2^* by undercutting P_1^* by η small. But $P_1^* - \eta \equiv P_2^* - \epsilon - \eta$, which contradicts the assumption that P_2^* is part of a Nash equilibrium.

Case 2: Suppose $P_2^* = \hat{P}$. Then firm 1's best response is to charge R, since $\pi_1(R) = (R-M)B > \pi_1(P_1)$ for any $P_1 \in [\hat{P}, R)$, given $P_2^* = \hat{P}$. But, since firm 1's best response to $P_2^* = \hat{P}$ is $P_1^* = R$, it pays firm 2 to deviate from $\hat{P}$ to $R - \epsilon$. This contradicts the assumption that $P_2^* = \hat{P}$ is part of a Nash equilibrium. We therefore conclude that there does not exist a pure strategy Bertrand-Nash equilibrium. □

The intuition behind the non-existence of a pure-strategy (or even an epsilon-) equilibrium is as follows. Under perfect competition (when product characteristics are unimportant), price competition drives prices towards marginal costs as firms compete for the price-conscious consumers. Under monopolistic competition, prices tend towards the reservation price as the firms attempt to exploit the monopoly

5 This obtains if, at $\hat{P}$, the firm gets all of the price-conscious consumers (in which case it sells $B+I$ units). If not, it either shares the price conscious consumers (and sells $B+I/2$ units) or loses the price-conscious consumers to the other firm (and sells B units).

power arising from differentiated products. In the hybrid case when both types of customers are present, these two forces repel one another in such a way that any fixed-price pair is unstable. Between $\hat{P}$ and R, firms want to undercut their rival to capture the price-conscious customers; but at $\hat{P}$ the incentive to exploit monopolistic power becomes dominant.

In order to solve for an equilibrium when there is a mix of price-conscious and brand-loyal customers, firms must 'mix' their prices. In other words, a firm must preclude its rival from exploiting its price strategy, thereby precluding the rival from systematically undercutting its price. This can be achieved by randomly selecting prices.

To formalize this idea, let $H(P)$ and $F(P)$ be the distribution functions used by the home and foreign firms to randomize prices. Intuitively, each firm draws from these distributions in order to set prices. While the general demand configurations given in equation 1 allow for 'ties,' if H and F are continuous, the probability that the firms will charge the same price is zero. If the foreign firm randomizes based on F and firm one sets a price of P_1, then with probability $[1 - F(P_1)]$ firm 1 sets the lowest price, and makes a profit $(P_1 - M)(B + I)$, and with probability $F(P_1)$ it sets the highest price and makes a profit of $(P_1 - M)B$. Firm 1 faces the problem of choosing H optimally:

$$\max_{dH} E\pi_1 = \int_{\hat{P}}^{R} [(P - M)(B + I)(1 - F(P)) + (P - M)BF(P)]dH(P). \tag{3}$$

The following proposition characterizes the Bertrand-Nash mixed-strategy equilibrium.

PROPOSITION 2. *Suppose the two firms have identical marginal costs ($M_1 = M_2 = M$), identical numbers of brand-conscious customers ($B_1 = B_2 = B > 0$), and that there are $I > 0$ price-conscious customers. Then there exists a unique mixed-strategy Bertrand-Nash equilibrium such that firms in each country choose a price at random from the distribution function, $G(P)$, defined by*

$$G(P) = 1 - \frac{(R - P)B}{(P - M)I} \text{ for } P \in [\hat{P}, R]. \tag{4}$$

Furthermore, expected profits are $E\pi_i = (R - M)B > 0$.

Proof. First note that $G(P)$ is a well-defined distribution function as (i) $G(\hat{P}) = 0$ and $G(R) = 1$; (ii) $dG/dP > 0$; and (iii) $G(P)$ is continuous. In order to verify that G satisfies the mutual best-response property, we shall show that, for the three intervals $[0, \hat{P})$, $[\hat{P}, R]$, and (R, ∞), it does not pay for a firm to use a randomizing device different from G, given that the other firm uses G.

Suppose firm 2 uses G to randomize prices. Then if firm 1 charges P_1, its expected profits are

$$E\pi_1(P_1) = \begin{cases} (P_1 - M)(B + I) & \text{if } P_1 < \hat{P} \\ (P_1 - M)(B + I)(1 - G(P_1)) + (P_1 - M)BG(P_1) & \text{if } P_1 \in [\hat{P}, R] \\ 0 & \text{if } P_1 > R. \end{cases} \tag{5}$$

As in proposition 1, firm 1 would never price below $\hat{P}$, since profits are lower than when R is charged. Firm 1 would never price above R, since profits are zero for such prices. For $P_1 \in [\hat{P}, R]$, note that equations (4) and (5) imply

$$E\pi_1 = (P_1 - M)(B + I)[1 - G(P_1)] + (P_1 - M)BG(P_1) = (R - M)B,$$

so that firm 1 earns the same expected profits for every $P_1 \in [\hat{P}, R]$. When $F = H = G$, equation (5) is constant for all $P_1 \in [\hat{P}, R]$ and lower for $P_1 \notin [\hat{P}, R]$. Consequently, firm 1 has no incentive to use a device other than G to select its price. By symmetry the same arguments hold for firm 2. Hence, each firm selecting a price at random from $G(P)$ comprises a Bertrand-Nash mixed-strategy equilibrium. Uniqueness follows by arguments similar to lemmas 12, 15, and theorem 1 in Baye, Kovenock, and de Vries (1992). □

We have shown that mixed strategies arise when one considers a mixture of the two commonly considered polar cases in trade theory. As the mixture of price-conscious and brand-loyal customers seems at least as plausible as the two extremes, we next focus on trade issues.

V. TRADE ISSUES

1. Free trade

Having established and constructed the equilibrium pricing strategies for the hybrid case, we now characterize the implications for trade. For simplicity, we maintain our assumptions that the firms have identical marginal costs ($M < R$), identical numbers of brand-loyal customers ($B > 0$), and that there are price-conscious customers ($I > 0$). In addition, we now assume the countries have identical populations ($B_{1f} + B_{2f} + I_f = B_{1h} + B_{2h} + I_h = D$, say). Our analysis considers the pattern of trade from the viewpoint of the home country, that is, firm 1.

The potential exports of the home firm consist of two components. One component is the certain delivery of B_{1f} units to the foreign brand-1-loyal customers, and the other component constitutes an uncertain element of trade. If the home firm happens to set the lowest price, then it serves the I_f price-conscious customers abroad. Given the symmetric nature of the equilibrium defined in proposition 2, each firm has a 50 per cent chance of charging the lowest price and thus of servicing the I price-conscious customers. Table 1 summarizes the possible trade configurations. From the table, it is easy to deduce that the home firm ships, on average, $B_{1f} + I_f/2$ units abroad. This represents the home country's average exports. Similarly, the volatility of the home firm's exports is Var [exports] $= (I_f)^2/4$. Hence, the greater the number of price-conscious customers abroad, the greater is the volatility of the home country's exports. Similar results obtain for the import flows. In summary, we have established the following proposition on the pattern of trade.

PROPOSITION 3. *In international markets where firms engage in price competition and there are brand-loyal and price-conscious customers, equilibrium pricing*

TABLE 1
The pattern of trade

Home exports	Home imports	Probability
B_{1f}	$B_{2h} + I_h$	1/2
$B_{1f} + I_f$	B_{2h}	1/2

strategies imply positive but stochastic cross-hauling. The two-way 'interindustry trade' component of trade is deterministic, whereas the one-way 'intra-industry trade' component is random. The volatility of trade increases with the number of price-conscious customers.

COROLLARY 1. *International markets with relatively few brand-loyal customers are more volatile (have a higher coefficient of variation) than markets with a relatively high content of brand-loyal customers.*

Proof. The coefficient of variation ($\mu \equiv \sqrt{\text{var}}/\text{mean}$) of country 1's exports is

$$\mu = \frac{1}{1 + 2B_{1f}/I_f}.$$

Clearly, μ decreases as the ratio of brand-loyal to price-informed customers, B_{1f}/I_f, increases. □

We next focus on the terms of trade. As trade involves similar commodities, define the terms of trade of the home country as the quantity of the import it receives in return for one unit of the export (assuming away trade in other products). In the case at hand, the terms of trade are stochastic, owing to the mixed pricing strategies.

In particular, from table 1 it follows that the terms of trade of the home country, TT_h, take on only two values with equal probability: $(B_{2h} + I_h)/B_{1f}$ and $B_{2h}/(B_{1f} + I_f)$. For simplicity, suppose the brand-loyal customers in each country are equally divided between the two products ($B_{1h} = B_{2h} = B_h$ and $B_{1f} = B_{2f} = B_f$). Given this, the expected terms of trade for the home country are

$$E[TT_h] = \frac{1}{2}\frac{B_h + I_h}{B_f} + \frac{1}{2}\frac{B_h}{B_f + I_f} \tag{6}$$

$$= \frac{(B_h + I_h)(B_f + I_f) + B_h B_f}{2B_f(B_f + I_f)}. \tag{7}$$

Suppose, $B_h > B_f$, while maintaining the assumption that the populations in the two countries are equal (to D) and maintaining all other symmetry assumptions (hence, $I_h < I_f$). In this instance, a larger fraction of the home population is brand loyal vis à vis the foreign population, and all other things are equal. We have the following result.

PROPOSITION 4. *Suppose the home country has a comparative brand-loyalty advantage, in the sense that a greater fraction of the population are brand loyal at home than abroad ($B_h > B_f$ and $I_h < I_f$). Then the home country enjoys the more favourable expected terms of trade: $E[TT_h] > E[TT_f]$.*

Proof. Using equation (7),

$$\frac{E[TT_h]}{E[TT_f]} = \frac{B_h(B_h + I_h)}{B_f(B_f + I_f)}. \tag{8}$$

Under the assumptions that the populations are equal ($B_{1f}+B_{2f}+I_f = B_{1h}+B_{2h}+I_h = D$), that the brand-loyal customers in each country are equally divided between the two products ($B_{1h} = B_{2h} = B_h$ and $B_{1f} = B_{2f} = B_f$), and that $B_h > B_f$, simple manipulation reveals $1/2 > \alpha > \beta$, where $\alpha \equiv B_h/D$ and $\beta \equiv B_f/D$. Furthermore

$$\frac{E[TT_h]}{E[TT_f]} = \frac{B_h(D - B_h)}{B_f(D - B_f)} = \frac{\alpha(1-\alpha)}{\beta(1-\beta)}.$$

But it is well known that this ratio is greater than unity if $1/2 > \alpha > \beta$. □

Proposition 4 reveals that the average terms of trade depend on the composition of the population in the two countries. In particular, the foreign firm wishes to ship more goods to the domestic market where customers care less about the price and more about the product characteristics, because it is better able to exploit its monopoly power in such a market. Effectively, the firms ship goods away from the market with the most price-conscious consumers. Therefore, in the hybrid model, the terms of trade provide more of a measure of comparative price consciousness (or brand consciousness) than of comparative cost advantage.

The general notion is that a reduction in monopoly power is socially desirable. In this sense, international trade by the duopolists can be considered welfare enhancing, since it reduces their otherwise monopolistic power. Clearly, in autarky, the firms (each a monopolist) will charge R to exploit the zero-one demand structure fully. As the equilibrium-randomized-pricing strategies have R as their upper limit, the consumers are unambiguously better off with trade than without trade.[6] However, under free trade, the duopolists experience lower expected profits than under autarky and are worse off; cf. Anderson et al. (1989, prop. 1). Given the zero-one demand structure and the assumed risk neutrality of agents, the monetary gain to consumers exactly offsets the loss to producers; trade results in a redistribution from producers to consumers, but the total surplus is left unchanged.

6 This is a direct implication of stochastic dominance.

2. The impact of subsidies

Finally, we examine the effects of lump-sum and specific subsidies.[7] Consider first the mutual imposition of a lump-sum subsidy (a specific subsidy unrelated to the actual quantity sold). Denote the lump-sum subsidies given to the two firms as $K_1 = K_2 = K$. Hence the cost functions (assuming identical marginal costs) become

$$C_i(Q) = MQ - K. \tag{9}$$

Importantly, the presence of K in equation (9) does not affect the lower bound price $\hat{P}$ in equation (2), nor the equilibrium distribution $G(P)$ in equation (4), provided profits at the reservation price are non-negative. In fact, the only effect of allowing for a K is to change expected profits by K. This conclusion is quite general and applies even if only one country gives a subsidy, or if K is negative (a lump-sum tariff or fixed costs).

In contrast to the case of a lump-sum subsidy, a specific subsidy of σ per unit does affect the mixed-pricing strategies. If both countries provide their firms with a production subsidy of σ (i.e., the case of a subsidy war), the effect is to lower marginal costs to $M^* \equiv M - \sigma$. Hence, this case is essentially equivalent to the previous analysis, except that one must substitute M^* for M in the relevant equations. The asymmetric case when only one country provides a subsidy is more intricate.

Suppose that the home country provides a unilateral production subsidy of $\sigma > 0$, so that the effective marginal cost of firm 1 is $M_1^* \equiv M - \sigma > 0$, while $M_2 = M$. Then, the home firm has a cost advantage over the foreign firm. In particular, the home firm can be assured of profits of $\pi_1^* = (R - M)B + \sigma(B + I)$ by pricing at $\hat{P}$. But an argument similar to the one in the proof of proposition 1 reveals that there does not exist a pure-strategy equilibrium. However, the following mixed-strategy Bertrand-Nash equilibrium does exist under a home production subsidy:

$$H(P) = 1 - \frac{(R - P)B}{(P - M)I}, \qquad \text{for } \in [\hat{P}, R], \tag{10}$$

$$F(P) = \begin{cases} 1 - \dfrac{(R - P)B + \sigma I}{(P - M_1^*)I}, & \text{for } P \in [\hat{P}, R) \\ 1, & \text{for } P = R. \end{cases} \tag{11}$$

We sketch the proof that this indeed constitutes an equilibrium. By proposition 2 it is immediate that $H(P)$ is well defined and that firm 2 has no incentive to deviate given that firm 1 plays $H(P)$. Further, $F(P)$ is a well-defined distribution function with a mass point of $\sigma/(R - M_1^*)$ at R. Substituting equation (11) into the expected profit formula of firm 1 reveals that $E\pi_1 = \pi_1^*$. Note that if firm 1 plays $H(P)$, the mass point in $F(P)$ at R plays no role, as firm 1 changes R with probability zero. It

7 The case of tariffs is somewhat similar; cf. Fisher and Wilson (1987).

follows that firm 1 has no incentive to use a device different from H to randomize prices; F and G comprise a Nash equilibrium.

Note that $H(P) > F(P)$ on the open interval $(\hat{P}, R)$, so that the home firm is more likely to charge lower prices than the foreign firm.[8] Hence, the home firm will capture the price-conscious consumers more frequently than the foreign firm. The jump in $F(P)$ at R implies that firm 2 charges R with positive probability. However, the probability that the two firms charge the same price is zero, since H is continuous at R. The subsidy increases home profits by $\sigma(B+I)$. To summarize, we state:

PROPOSITION 5. *When differences in marginal costs exist (owing to a unilateral production subsidy or technological advantage), the equilibrium mixed strategies are such that the firm with the lowest marginal cost charges lower prices more frequently than the firm with the highest marginal cost.*

COROLLARY 2. *Suppose that the terms of trade are as in equation (6) and that the home firm is given a specific subsidy. Then the terms of trade for the home country are reduced, because the home firm is more likely to service the price-conscious consumers.*

Proof. As $H(P) > F(P)$ on $(\hat{P}, R)$, the coefficient of $B_h/(B_f + I_f)$ in equation (6) increases above 1/2, while the coefficient on the other term becomes less than 1/2.□

One can generalize the above analysis to examine the impact of a specific tariff. Such a tariff drives a wedge between a firm's domestic price and foreign price, and thus the two markets become segmented. The analysis of such a model would be similar to that described above, except that one must allow for different pricing strategies in each market.

VI. CONCLUSION

We began by investigating stylized versions of two polar models of trade which are commonly considered in the literature: perfect competition and monopolistic competition. As is well known, for both cases there exist pure strategy equilibria. We then examined the intermediate case, where some consumers view foreign and domestic products as homogeneous (and thus care only about price), while others have a preference for the product of a particular country. This hybrid model is motivated by the observation that, in many markets, some consumers view products as homogeneous while others view products as heterogeneous. Some consumers have a preference for French wine, some a preference for California wine, while still others choose between the two purely on the basis of price. Our analysis reveals that, in such instances, a pure-strategy equilibrium does not exist. The reason is that competition for price-conscious customers tends to drive prices towards

8 This is a stochastic dominance condition.

marginal costs, while incentives regarding brand-loyal customers exert the opposite influence, as firms attempt to exploit the monopolistic power generated by perceived product heterogeneity. The equilibrium response to these opposing forces is to randomize prices, thus preventing the rival from systematically undercutting the price. Casual observation of the transatlantic airline market, which was mentioned in the introduction, suggests that this randomization is consistent with the data.

As a consequence of the equilibrium strategies, prices on the international markets are stochastic. Because this type of 'endogenous uncertainty' had not been investigated in the trade literature, we examined the implications for trade of the mixed-strategy equilibrium. We showed that a country's comparative advantage is related to the price consciousness of its customers, that the terms of trade are stochastic, and that trade flows may be decomposed into deterministic and random components. Finally, we showed that the presence of lump-sum subsidies does not alter the nature of the equilibrium, but that specific (per unit) subsidies do. In equilibrium, the firm that enjoys a production subsidy will charge lower prices more frequently than its competitor. The impact of other types of trade policies in this type of model remains for future research.

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