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The incidence of overdissipation in rent-seeking contests *

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Abstract. Tullock's analysis of rent seeking and overdissipation is reconsidered. We show that, while equilibrium strategies do not permit *overdissipation in expectation*, for particular realizations of players' mixed strategies the total amount spent competing for rents can exceed the value of the prize. We also show that the cross-sectional *incidence of overdissipation* in the perfectly discriminating contest ranges from 0.50 to 0.44 as the number of players increases from two to infinity. Thus, even though the original analysis of overdissipation is flawed, there are instances in which rent-seekers spend more than the prize is worth.

1. Introduction

Gordon Tullock's seminal contribution in the area of rent-seeking, and wasteful over-dissipation in particular, has not had the influence outside of *Public Choice* that it deserves. The purpose of this paper is to point out that even though his original analysis of overdissipation is technically flawed, the definition of overdissipation can be modified to explain instances in which rational rent-seekers spend more to win a prize than the prize is worth.

Specifically, since Tullock's seminal paper in 1967 most of the literature has focused on the degree to which the competition for rent dissipates that rent. While this literature extends across several fields,[1] it is concentrated to a large extent in the field of public choice,[2] where a standard tool in the theoretical analysis of rent-seeking is Tullock's rent-seeking game (1975, 1980). In this game, n risk-neutral players enjoy complete information and simultaneously submit nonnegative bids for a prize worth Q dollars. Letting $(x_1, \ldots, x_n) \geq 0$ denote the bids of players 1 through n, the probability player 1 wins the prize is given by

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$$p_i(x_1, \ldots, x_n) = \begin{cases} 1/n & \text{if } x_1 = x_2 = \ldots = x_n = 0, \\ \frac{x_i^R}{\sum_{j=1}^n x_j^R} & \text{otherwise.} \end{cases}$$

Here R is a parameter, $R > 0$. If $R = \infty$, then the game becomes perfectly discriminating and coincides with the all-pay auction. The payoff to player i from submitting a bid of x_i when the other n-1 players submit bids of $x_{-i} \equiv (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots x_n)$ is given by

$$U_i(x_i, x_{-i}) = p_i(x_i, x_{-i})Q - x_i.$$

Henceforth, we will refer to this symmetric, simultaneous-move game of complete information as the *Tullock game*.

By the end of the 1970s, two competing postulates emerged about rent-seeking games:

Posner's Rent Dissipation Postulate: In equilibrium, the total expenditures of rent-seekers equals the value of the prize.

Tullock's Rent Dissipation Postulate: In equilibrium, rent-seeking expenditures exceed the value of the prize when $R > n/(n - 1)$.

Posner's postulate (1975) relies on a strong free-entry assumption: If existing rent-seekers were in the aggregate spending less than the value of the prize, their expected profits would be positive. This would induce entry by other rent-seekers until profits are driven to zero. Tullock's postulate is based on the Tullock game and his observation that when $R > n/(n-1)$, the sum of the solutions to each player's first order conditions exceeds the value of the prize Q. Tullock (1980, 1984, 1985, 1987, 1989) devoted considerable attention to this possibility, presumably because of the strong implication for excessive social waste.[3] Indeed, as R increased, the amount of overdissipation would tend to infinity. A large literature emerged in an attempt to eliminate the apparent overdissipation of rents by altering the Tullock game.[4] Contributions in this line of research include Corcoran (1984), Corcoran and Karels (1985), Higgins, et al. (1987), Michaels (1988), Allard (1988), Leininger (1993), Leininger and Yang (1994) and Ellingsen (1991).

It is now widely recognized (Hillman and Samet (1987), Baye, et al. (1989, 1993, 1994, 1996)) that expected overdissipation is not part of a Nash equilibrium to the Tullock game for any value of R, even as R approaches infinity. The reason is that the Tullock game has a pure strategy Nash equilibrium if and only if $R \leq n/(n-1)$. For $R > n/(n-1)$ the symmetric solution to the players' first order conditions for expected payoff maximization does not yield a global maximum; at this solution players have a negative expected payoff, which is dominated by bidding zero. Thus, Tullock's postulate is based on a false premise.

In light of this, it is perhaps surprising that we demonstrate below that the *overdissipation postulate* for which Tullock is most frequently criticized can, in fact, be defended on theoretical grounds within the confines of his original model! The defense, it turns out, relies on the fact that when the Tullock parameter exceeds n/(n–1) the Nash equilibrium involves mixed-strategies. Specifically, Baye, Kovenock, and de Vries (1994) show that equilibrium mixed strategies in the Tullock game do not permit *overdissipation in expectation*: the expected total amount spent competing for rents cannot exceed the value of the prize.[5] However, since the equilibrium involves mixed-strategies, it turns out that for particular realizations of the mixed strategies the total amount spent competing for rents can exceed the value of the prize! In fact, we show below that the cross-sectional *incidence of overdissipation* may be quite high. For a symmetric perfectly discriminating contest ($R = \infty$), the probability of overdissipation in a symmetric equilibrium is roughly one-half, ranging from exactly one-half in the two player case to approximately .44 as the number of players approaches infinity.

The implication of this is straightforward: even when rent-seekers have complete information and are "perfect calculators", roughly one-half of the time they will spend more in the aggregate than the prize is worth. Roughly equally frequently they will, as a group, spend less than it is worth. This stochastic nature of the overdissipation of rents when $R > n/(n - 1)$ is the conceptual innovation that we examine in Sections 2 and 3 of this paper.

2. Defining the overdissipation of rents

For the two player case, Tullock postulated that equilibrium entails the overdissipation of rents when $R > 2$. As we noted in the introduction, in this case the only Nash equilibria to the Tullock game are in nondegenerate mixed-strategies. For this reason, it is necessary to distinguish between the expected level of rent dissipation that arises based on the ex ante strategies employed by players, and the level of rent dissipation that arises ex post (that is, for particular realizations of the strategies). In addition, it is useful to distinguish situations where the group as a whole spends more than the value of the prize (either in an ex ante or ex post sense) from those in which one or more individuals each spend more than the value of the prize. This gives rise to four alternative notions of the *overdissipation* of rents:

(EIO) **Expected Individual Overdissipation** occurs if an individual player's expected bid exceeds the value of the prize.

(EAO) **Expected Aggregate Overdissipation** occurs if the expected sum of the payments by the players exceeds the value of the prize.

(PIO) **Probabilistic Individual Overdissipation** occurs if there is a positive probability that an individual player bids more than the value of the prize.

(PAO) **Probabilistic Aggregate Overdissipation** occurs if there is a positive probability that the sum of all players' bids exceeds the value of the prize.

The following result is immediate and shows the relation among these four definitions of overdissipation.

Proposition 1: For the Tullock game,
(a) EIO $\Rightarrow$ EAO $\Rightarrow$ PAO;
(b) EIO $\Rightarrow$ PIO $\Rightarrow$ PAO.

Thus, for the Tullock game, the broadest of the definitions of overdissipation is PAO, and the most narrow is EIO. Notice that the contrapositive of Proposition 1 implies that if there is not probabilistic aggregate overdissipation, then there is not overdissipation in the other three senses either.

3. Equilibrium overdissipation in the Tullock model?

We begin with

Proposition 2: There do not exist equilibria to the Tullock game in which EAO, EIO, or PIO arise.

The formal proof of this proposition merely involves extending the results in Baye, Kovenock and de Vries (1994) from the two player case to the n-player case, and is thus omitted. The essential intuition can be seen by noting that a player can guarantee a payoff of at least zero by bidding zero. Hence, no equilibrium strategy can involve PIO, since bids above Q guarantee a negative payoff, and hence are strictly dominated. Similarly, no equilibrium can involve EIO because EIO requires PIO. Finally, since the above argument implies that $U_i(x_i, x_{-i}) \geq 0$ for every i, summing over all players and noting that the prize is awarded with probability one implies that no equilibrium can involve EAO. Furthermore, these arguments are valid for both pure and nondegenerate mixed-strategy equilibria.

Interestingly, however, the Tullock game does exhibit probabilistic aggregate overdissipation when R exceeds n/(n-1):

Proposition 3: Suppose $R > n/(n-1)$. Then in any Nash equilibrium to the Tullock game, PAO arises.

Proposition 3 follows directly from that fact that equilibria to Tullock's original game involve non-degenerate mixed-strategies if $R > n/(n-1)$. It indicates that an incidence of aggregate overdissipation is indeed possible in the original Tullock framework, but only in those instances where the equilibrium involves nondegenerate mixed-strategies and when one looks at *aggregate ex post* expenditures. We will illustrate that the actual incidence of overdissipation due to PAO can be quite high.

To this end we will use as a benchmark the perfectly discriminating (Hillman and Riley (1989)) or first-price all-pay auction (Baye et al. (1989, 1993, 1996)) version of the Tullock game. This game form is the limiting case of the Tullock game when $R = \infty$; thus, the probability that player i wins the prize is one if player i submits the highest bid and zero otherwise.[6] The $R = \infty$ case is a useful benchmark because Baye, Kovenock, and de Vries (1993) have shown that at this level of R the expected level of rent dissipation is maximized. Furthermore, when $R = \infty$ there is complete rent dissipation in the sense of EAO (the expected sum of the bids exactly equals the value of the prize).

Without loss of generality, suppose the value of the prize, Q, equals 1. Hence the payoff to player i as a function of the vector of bids of all n players is

$$U_i(x_1, x_2, \ldots, x_n) = \begin{cases} -x_i & \text{if } \exists j \text{ such that } x_j > x_i \\ \frac{1}{m} - x_i & \text{if i ties for high bid with } m-1 \text{ others} \\ 1 - x_i & \text{if } x_i > x_j \ \forall j \neq i. \end{cases}$$

For n = 2 the unique equilibrium of this game is symmetric (Hillman and Riley, 1989). For $n > 2$ there is a continuum of asymmetric equilibria as well as a unique symmetric equilibrium (Baye, Kovenock and de Vries, 1996). We focus here on the symmetric mixed-strategy equilibrium, which involves players randomizing according to a continuous mixed strategy with associated cumulative distribution function $F(x) \equiv x^{1/(n-1)}$ on [0,1]. This symmetric equilibrium fully dissipates rents in the sense of EAO, as do all of the asymmetric equilibria (Baye, Kovenock, and de Vries, 1996).

Since each player's bid is a random draw from F and the value of the prize is one, there cannot be overdissipation in the sense of either EAO, EIO, or PIO (this illustrates Proposition 2). However, notice that there is a positive probability that the sum of the realizations of the players' bids exceeds 1, i.e. the assumed value of the prize.

To see this, let $z = x_1 + x_2 + \ldots + x_n$ denote the sum of the bids. Notice that z is a random variable induced by the mixed strategies employed by the players, so let $G(Z) = \text{Prob}\{z \leq Z\}$ be its cumulative distribution function. The probability of overdissipation is given by the probability that the sum of the bids exceeds unity, which is $\text{Prob}\{z > 1\} = 1 - G(1)$. The symmetric Nash equilibrium mixed strategies imply that each x_i has a density $f(x_i) = a{x_i}^{a-1}$ on [0, 1], where a = 1/(n–1). Hence,

$$G(1) = a^n \int_0^1 x_n^{a-1} \int_0^{1-x_n} x_{n-1}^{a-1} \ldots \int_0^{1-x_n-x_{n-1}\ldots-x_2} x_1^{a-1} dx_1 dx_2 \ldots dx_n. \quad (1)$$

We now state the following general result:

Proposition 4. Suppose $R = \infty$ and $n \geq 2$ in the original simultaneous-move Tullock game. Then in any symmetric equilibrium we have PAO $\in$ [0.44, 0.5]. More specifically:

a. The probability of aggregate overdissipation is

$$1 - G(1) = 1 - \left(\frac{n-1}{n}\right)\left[\Gamma\left(\frac{n}{n-1}\right)\right]^{n-1}. \quad (2)$$

where $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$ is the *Gamma function*;

b. The probability of aggregate overdissipation is monotonically decreasing in the number of players;

c. The probability of aggregate overdissipation is maximized in the two player case in which the probability of aggregate overdissipation is exactly ½;[7]

d. In the limit as the number of players tends to infinity, the probability of aggregate overdissipation tends to $1 - e^{-\gamma} \approx .44$, where γ is Euler's constant.[8] Hence, the probability of aggregate overdissipation is bounded from below by .44.

Proof: See the Appendix.

Proposition 4 makes it clear that Tullock's postulate is, in a sense, correct: in the aggregate, rent-seekers may frequently spend more to win a prize than the prize is worth. This result helps explain why PAO is confirmed in some of the published experimental literature (cf. Millner and Pratt, 1989, 1990) as well as in the experiments we have run at the CREED laboratory, see Potters, de Vries and Van Winden (1997). The theoretical and empirical literatures can be reconciled because when $R > n/(n-1)$, equilibria of the Tullock game involve nondegenerate mixed-strategies.

It is worthwhile to provide some intuition for the contents of Proposition 4. In the two player case, it is easy to see why the incidence of aggregate overdissipation must be positive: with probability $1/2 \cdot 1/2 = 1/4$ both players bid more than $1/2$. As the number of players increases, players begin submitting low bids more frequently than higher ones. The increase in the number of players is not sufficient to offset this effect, and thus the incidence of aggregate overdissipation falls as the number of players rises. It is not too difficult to see why the incidence remains strictly within the interval (0, 1) as the number of players n increases. If the incidence of aggregate overdissipation tended to 1 as the number of players approached infinity, then in the limit the sum of the bids would exceed the value of the prize with probability one. This would violate individual rationality, since each player is guaranteed a payoff of at least zero in any Nash equilibrium. A similar arbitrage argument precludes the incidence from converging to zero as the number of players goes to infinity.

4. Other rent-seeking contests

It is of interest to apply our new definitions of overdissipation to other types of contests. This will also make it clear why it is useful to make a distinction between individual and aggregate overdissipation.

Consider, for instance, the (stationary) symmetric equilibrium of the two person symmetric war of attrition, which is equivalent to a second-price all-pay auction. Assume that the value of the prize and the cost to each player per unit of time spent fighting both equal one, and that there is no discounting. Each player's symmetric equilibrium strategy in this game is mixed and is represented by the cdf, $G(t) = 1 - e^{-t}$. This gives the probability that the player stops fighting before t. In the interpretation as a second-price all-pay auction it is the probability the player bids below t. The game ends at the minimum realized stopping time. The time elapsed equals the cost incurred by each player. Hence, the distribution of the cost incurred by each player fighting for the prize is the distribution of the minimum order statistic $G_{min}(t) = 1 - [1 - G(t)]^2 = 1 - e^{-2t}$, which also gives the distribution of the game's termination time.

The expected payoff to each player in this equilibrium is zero. Since each player incurs a cost of 1 per unit of time until the game ends, if the game stops after t = ½, the aggregate cost of the contest will exceed the value of the prize; overdissipation will arise. Hence, PAO characterizes the symmetric equilibrium. Aggregate overdissipation occurs with probability $1 - G_{\min}(1/2) = e^{-1} \approx .368$, which is lower than in the first-price all-pay auction.

The war of attrition, unlike the first-price all-pay auction, also has the property that the probability of overdissipation by a *single* player is nonzero, i.e. PIO is also a property of the symmetric equilibrium. In this equilibrium, if the game ends after t = 1 *each* player's individual cost is greater than the value of the prize. This event has probability $1 - G_{\min}(1) = e^{-2} \approx .135$.

This makes it clear that Tullock's original postulate is, in a way, correct; every individual player can make payments that are greater than the value of the prize. Individual overdissipation, however, cannot occur in the game Tullock examined (in which all players pay their own bids) or in the manner Tullock described (*ex ante* expectation).

Another rent-seeking contest in which it is possible for individual players to make payments greater than the value of the prize is the sad-loser auction. In this auction each player i simultaneously bids x_i. The highest bidder wins the prize (of value 1) and is refunded her bid. The remaining bidders forfeit their bids.

This type of auction might be viewed as a reduced-form for situations, such as contests for procurement contracts, in which the bids represent pre-award (prototype) development costs, and the winning bidder can recoup these costs under the terms of the contract. In the two bidder case the individual players' symmetric equilibrium bidding strategies are $G(x) = x/(1 + x)$, which has an unbounded mean. Each player earns a zero payoff in expectation. The aggregate payment in the game is the minimum order statistic, which has a distribution $G_{\min}(x) = 1 - [1/(1 + x)]^2$. Hence the probability of aggregate overdissipation is $1 - G_{\min}(1) = 1/4$. Since only the loser pays, this is also the probability that there is individual overdissipation by some player. Hence, PIO and thus also PAO characterize the symmetric equilibrium to this game.

Like the first price all-pay auction, the war of attrition and the sad loser auction are contests with complete information. Of course, if we allow for incomplete information it should not seem surprising that overdissipation is possible, and that the probability of overdissipation will depend on the distributional assumptions maintained when transforming the game to one of complete, but imperfect information in which types are chosen by nature. Due to the lack of a clear benchmark model, we omit a formal analysis at this stage.

5. Conclusion

Gordon Tullock has done the profession a great service by pointing to the need to understand how institutions affect the wasteful expenditure of resources on rent seeking. Tullock's (1980) description of rent seeking through a contest has become the industry's standard. This contest does not have a pure strategy equilibrium for a range of the exponent parameter R, but a mixed strategy solution exists. Initially, this was not well understood and led to some speculation that there might be overdissipation. As a response[9] several interesting perturbations of the game, including versions with a sequential move structure, risk aversion and entry, have been investigated. But as we have made clear (see Baye, Kovenock and de Vries (1994)), the Nash concept never leads to overdissipation in an expected sense as long as individuals have the chance to opt out of the game and receive a payoff of zero (by spending zero).

Even though the Nash concept precludes *expected* overdissipation in the Tullock game, for particular realizations of the players' mixed strategies, aggregate expenditures may exceed the value of the prize. In this paper we therefore introduced the concept of the incidence of overdissipation, and calculated this incidence for some of the standard contests used for modelling rent-seeking behavior. For the perfectly discriminating version of Tullock's original game, we showed that an increase in the number of players lowers the incidence, but it never drives it down to zero.

In light of the results in Section 3, we believe Tullock should reassess his distaste for mixed-strategy Nash equilibria. There are well-reasoned justifications of mixed-strategy Nash equilibria appearing in the literature (see, for instance, Brandenburger (1992)). For those like Tullock who are searching for a justification for overdissipation, it would seem that a powerful rationale for using mixed-strategies is that they can generate an incidence of overdissipation.[10]

Otherwise, one must not only propose an alternative solution concept for his game, but an alternative justification for the overdissipation of rents. Mixed strategies, as it turns out, provide both the needed solution concept as well as a defense for the Tullock postulate.

In concluding, we note that there are solution concepts that can rationalize overdissipation in expectation. For instance, it is possible to find rationalizable strategies that yield overdissipation in expectation. However, these strategy choices cannot constitute a Nash equilibrium and, hence, if the choices or conjectures generating them were mutual knowledge, at least one player would not be playing rationally. Likewise, overdissipation in expectation can arise in ϵ-equilibria to the Tullock game. However, when Gordon Tullock (1989) claims that the theory of efficient rent seeking is "based on the theory that people are perfect calculators", and dismisses experimental work for relying on the computational ability of MBA students who clearly "are not making correct calculations", it is clear that he has a Nash-like consistency requirement in mind. Section 3 above shows that *incidences* of aggregate overdissipation are not at odds with Nash consistency. Tullock's postulate is indeed correct: perfectly rational individuals might spend more, in the aggregate, competing for a prize than it is worth. But Tullock was right for the wrong reason.

Notes

1. See, for instance Posner (1975) and Fudenberg and Tirole (1987) in industrial organization, Krueger (1974) and Bhagwati (1982) in international trade, and Linster (1993) in the analysis of international alliances.
2. See surveys by Brooks and Heijdra (1989), Nitzan (1994), or Rowley (1991).
3. Tullock (1989) noted regarding the overdissipation result ... "when I demonstrated that perfect calculation leads to decidedly odd results even in a competitive market with free entry, I astonished myself". He went on to note that the original (1980) paper "was rejected by the *Journal of Political Economy* on the argument that it could not possibly be true that a competitive market would reach these results". In explaining why experiments run with MBA students for n = 2 and R = 3 did not yield overdissipation on average he reasoned "it is clear that the people concerned are not making correct calculations", and "it seems to me that ... these people do not understand the game".
4. In addition, numerous studies focus on the special case where R = 1 (see, for instance, Nitzan (1991); Paul and Wilhite (1991)). In this case, the solution to the first-order conditions do indeed yield a Nash equilibrium, but there is not overdissipation in the corresponding equilibrium.
5. Baye, Kovenock and de Vries (1994) analyze the case of n = 2 and $R > 2$. The method of proof is similar for $n > 2$ and $R > n/(n-1)$.
6. In the case of a tie among m players for the highest bid, each has a probability (1/m) of winning the prize.
7. For n = 3 this probability is $1 - \pi/6 \approx 0.48$, while for n = 4 it is approximately 0.466.
8. See Abramovitz and Stegun (1965). Formally, γ is defined by

$$\gamma = \lim_{h\to\infty}\left[1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{h} - \log h\right].$$

9. The working paper version of this paper (Baye, Kovenock, and de Vries, 1997) contains a more detailed response to Tullock's (1995) comments.
10. Recent work by Che and Gale (1996) shows that the symmetric equilibrium mixed-strategies that we identify are identical to the pure strategy bidding functions that arise when rent-seekers face budget constraints and incomplete information about the size of rivals' budgets.

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Appendix

To evaluate the multiple integral in equation (1) of the text, we use the following lemma:

Lemma A1: Let $\varphi(w) \equiv \int_0^{1-w} x^r(1-w-x)^s\,dx$. Then $\varphi(w) = (1-w)^{r+s+1}\beta(r+1,\ s+1)$, where $\beta(r+1,\ s+1) \equiv \int_0^1 x^r(1-x)^s\,dx$ is the *Beta function*.
Proof:

$$\begin{aligned}\varphi(w) &\equiv \int_0^{1-w} x^r(1-w-x)^s\,dx\\ &= (1-w)^{r+s+1}\int_0^1 t^r(1-t)^s\,dt\\ &= (1-w)^{r+s+1}\beta(r+1, s+1)\end{aligned}$$

by the change of variable $t = x/(1-w)$. **Q.E.D.**

Proof of Proposition 4:

(a) Applying the Lemma (taking $w = 1 - x_n - x_{n-1} - \ldots - x_2$, $r = a-1$, $s = 0$, and $x = x_1$) gives us

$$G(1) = a^n \int_0^1 x_n^{a-1} \int_0^{1-x_n} x_{n-1}^{a-1} \ldots \int_0^{1-x_n-\ldots-x_3} x_2^{a-1}(1-x_1-\ldots-x_2)^a$$
$$\beta(a, 1)dx_2dx_3\ldots dx_n.$$

Again, using the Lemma (taking $w = 1 - x_1 - \ldots - x_3$, $r = a-1$, $s = a$, and $x = x_2$) gives

$$G(1) = a^n\beta(a, 1)\beta(a, a+1)\int_0^1 x_0^{a-1}\ldots\int_0^{1-x_n-\ldots-x_4}$$
$$x_3^{a-1}(1-x_1-\ldots-x_3)^{2a}dx_3\ldots dx_n,$$

and with $w = 1 - x_1 - \ldots - x_4$, $r = a-1$, $s = 2a$, and $x = x_3$,

$$G(1) = a^n\beta(a, 1)\beta(a, a+1)\beta(a, 2a+1)\int_0^1 x_n^{a-1}\ldots\int_0^{1-x_n-\ldots-x_3}$$
$$x_4^{a-1}(1-x_n-\ldots-x_4)^{3a}dx_4\ldots dx_n.$$

Continuing in this fashion we obtain

$$\begin{aligned}G(1) &= a^n\beta(a, 1)\beta(a, a+1)\beta(a, 2a+1)\beta(a, 3a+1)\ldots\int_0^1 x_n^{a-1}(1-x_n)^{(n-1)a}dx_n\\ &= a^n\beta(a, 1)\beta(a, a+1)\beta(a, 2a+1)\beta(a, 3a+1)\ldots\beta(a, (n-1)a+1).\end{aligned}$$

Note that $\beta(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$, where $\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}dx$ is the *Gamma function*. Hence,[11]

$$\begin{aligned}G(1) = &\ a^n \frac{\Gamma(a)\Gamma(1)}{\Gamma(a+1)}\frac{\Gamma(a)\Gamma(a+1)}{\Gamma(2a+1)}\frac{\Gamma(a)\Gamma(2a+1)}{\Gamma(3a+1)}\ldots\frac{\Gamma(a)\Gamma((n-1)a+1)}{\Gamma(na+1)}\\ &= \frac{a^n[\Gamma(a)]^n}{\Gamma(na+1)} = \frac{[a\Gamma(a)]^n}{\Gamma(na+1)} = \frac{[\Gamma(a+1)]^n}{\Gamma(na+1)} = \left(\frac{1}{a+1}\right)[\Gamma(a+1)]^{n-1}.\end{aligned}$$

11. In these calculations we make use of the identities $\Gamma(t+1) = t\Gamma(t)$ and $\Gamma(n+1) = n!$, for t positive real and n positive integer valued.

Using the definition of a gives the form in equation (2) of the text.

(b) To establish monotonicity, let $P(a) = \frac{1}{1+a}[\Gamma(1+a)]^{1/a}$, where $a \in [0, 1]$, $a \equiv a(n) = 1/(n-1)$, and $n-1$ denotes number of opponents. It is sufficient to show that P(a) is decreasing in a, for this implies the probability of overdissipation, $1 - P(a(n))$, is increasing in n. Taking logs gives

$$\log P(a) = -\log(1+a) + \frac{1}{a}\log \Gamma(1+a).$$

Differentiating this expression yields

$$\frac{d \log P(a)}{da} = -\frac{1}{1+a} - \frac{1}{a^2}\log \Gamma(1+a) + \frac{1}{a}\Psi(a+1)$$

where Ψ is the Psi (or Digamma) function (see Abramowitz and Stegun, 1965). Limiting values are obtained by l'Hôpital's rule:

$$\frac{1}{a^2}[\log \Gamma(1+a) - a\Psi(1+a)]|_{a=0} =$$
$$\frac{\Psi(1+a) - \Psi(1+a) - a\Psi'(1+a)}{2a}|_{a=0} =$$
$$-\frac{1}{2}\Psi'(1+a)|_{a=0} = -\frac{1}{2}\Psi'(1) < 0$$

Hence

$$\frac{d \log P(a)}{da}|_{a=0} = -1 - \frac{1}{2}\Psi'(1) \approx -1.82 < 0$$

$$\frac{d \log P(a)}{da}|_{a=1} = -\frac{1}{2} - [\log \Gamma(2) - \Psi(2)] = -\frac{1}{2} - [0 - .42] = -.08 < 0.$$

To evaluate the intermediate values we write

$$\frac{d \log P(a)}{da} = \frac{1}{a^2}\left\{-\frac{a^2}{1+a} - \log \Gamma(1+a) + a\Psi(1+a)\right\} \tag{A1}$$

and analyze the sign of the terms within the curled brackets on the interval (0,1). Differentiating the term in brackets yields

$$-\frac{2a}{1+a} + \frac{a^2}{(1+a)^2} - \Psi(1+a) + \Psi(1+a) + a\Psi'(1+a) =$$
$$a\Psi'(1+a) + \frac{a^2 - 2a - 2a^2}{(1+a)^2} =$$
$$a[\Psi'(1+a) - \frac{2+a}{(1+a)^2}] =$$
$$a[\Psi'(1+a) - \frac{1}{1+a} - \frac{1}{(1+a)^2}]$$

We can concentrate on the term inside the last pair of square brackets, and write $y = 1 + a$, $y \in (1, 2)$, so that the derivative of the bracketed term becomes:

$$\Psi'(y) - \frac{1}{y} - \frac{1}{y^2}$$

We will show that this term does not change sign as $n \geq 2$ increases. Toward this end, note that

$$\Psi'(y) \equiv \int_0^\infty \frac{t\, e^{-yt}}{1 - e^{-t}} dt$$

Now from the Taylor expansion of the exponential

$$\frac{1}{1-e^{-t}} = \frac{e^t}{e^t-1} = \frac{1+t+t^2/2+t^3/6+\ldots}{t+t^2/2+t^3/6+\ldots} = 1 + \frac{1}{t+t^2/2+t^3/6+\ldots}$$

Hence, for $t \geq 0$:

$$\frac{t}{1 - e^{-t}} = t + \frac{1}{1 + t/2 + t^2/6 + \ldots} \leq t + \frac{1}{1 + t/2} \leq t + \frac{1}{1 + t/2} \leq t + 1$$

Thus

$$\begin{aligned} \Psi'(y) &< \int_0^\infty (t + 1)e^{-yt}dt \\ &= \int_0^\infty te^{-yt}dt + \int_0^\infty e^{-yt}dt \\ &= \frac{1}{y}\int_0^\infty tye^{-yt}dt + \frac{1}{y}\int_0^\infty ye^{-yt}dt \\ &= \frac{1}{y^2} + \frac{1}{y} \end{aligned}$$

and therefore

$$\Psi'(y) - \frac{1}{y} - \frac{1}{y^2} \leq 0.$$

Now the proof of monotonicity of $P(a)$ for $a \in [0, 1]$ is complete as

$$\frac{d \log P(a)}{da} = \frac{1}{a^2}\theta(a)$$

where $\theta(a)$ is the expression between curled brackets in (A1). At the end points

$$\theta(0) = 0, \theta(1) = -0.08 < 0$$

while the above analysis showed

$$\theta'(a) < 0 \text{ on } (0, 1] \text{ and } \theta'(0) = 0$$

Combining these gives

$$\theta(a) < 0 \text{ on } (0, 1]$$

Moreover,

$$1/a^2 > 0 \text{ on } (0, 1]$$

so that

$$\frac{d \log P(a)}{da} = \frac{1}{a^2}\theta(a) < 0 \text{ on } (0, 1]$$

Hence $\log P(a)$ is decreasing on $(0, 1]$. Incidently, note that

$$\log P(0) = \Psi(1) = \gamma \approx -.57$$

$$\log P(1) = -\log 2 \approx -.69$$

(c) This part follows directly from part (b).

(d) Setting h = n - 1, it follows that

$$\lim_{h\to\infty} \log\left[\Gamma\left(1+\frac{1}{h}\right)\right]^h = \lim_{h\to\infty}\left[\log\Gamma\left(1+\frac{1}{h}\right)\right]/(1/h),$$

which by l'Hôpital's rule equals

$$\lim_{h\to\infty}\left(\frac{\partial\Gamma(\omega)}{\partial\omega}/\,\Gamma(\omega)\right)|_{\omega=1+\frac{1}{h}}.$$

The expression $\frac{\partial\Gamma(\omega)}{\partial\omega}/\,\Gamma(\omega)$ is the Psi or Digamma function (see Abramowitz and Stegun (1965)) which, when evaluated at $\omega = 1$ is equal to $-\gamma$, where $\gamma \approx .5772$ is Euler's constant. Hence, as h (and hence n) goes to infinity

$$G(1) = \lim_{h\to\infty}\left(\frac{h}{h+1}\right)\left[\Gamma\left(\frac{h+1}{h}\right)\right]^h = e^{-\gamma} \approx .5615.$$

Therefore, in the limit, the probability of overdissipation is approximately 0.4385.