

1-1-1994

# Stylized Facts of Nominal Exchange Rate Returns

Casper G. de Vries  
*Erasmus University Rotterdam*

K. U. Leuven  
*Erasmus University Rotterdam*

Follow this and additional works at: <http://docs.lib.purdue.edu/ciberwp>

---

de Vries, Casper G. and Leuven, K. U., "Stylized Facts of Nominal Exchange Rate Returns" (1994). *Purdue CIBER Working Papers*. Paper 79.  
<http://docs.lib.purdue.edu/ciberwp/79>

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact [epubs@purdue.edu](mailto:epubs@purdue.edu) for additional information.

# **STYLIZED FACTS OF NOMINAL EXCHANGE RATE RETURNS**

**Casper G. de Vries\***  
**K. U. Leuven**  
*Erasmus University Rotterdam*

**94-002**

\*Casper de Vries was a CIBER Visiting Professor at Purdue University in the Spring, 1994

**Center for International Business Education and Research**  
**Purdue University**  
**Krannert Graduate School of Management**  
**1310 Krannert Building**  
**West Lafayette, IN 47907-1310**  
**Phone: (317) 494-4463**  
**FAX: (317) 494-9658**

STYLIZED FACTS OF NOMINAL EXCHANGE RATE RETURNS\*

Casper G. de VRIES

K.U.Leuven

and

Erasmus University Rotterdam

Fall 1991

Revised Summer 1992

ABSTRACT

This survey collects the stylized facts on nominal foreign exchange rate returns. The most salient statistical regularities: unit roots, fat tails, and volatility clusters are extensively discussed.

---

\* Prepared for the Handbook of International Macroeconomics, edited by F. van der Ploeg. This review is based on lecture notes for the MA class in Advanced International Monetary Economics at the K.U.Leuven. I am grateful to Filip Abraham, Hans Dewachter, Jürgen von Hagen, Kees Koedijk, Luc Lauwers, Charles van Marrewijk, Theo Nijman, Rick van der Ploeg, Peter Schotman, Philip Stork, Guy Van Camp and Jean-Marie Viaene who all gave valuable comments on an earlier version of the paper. I also like to thank the students who participated in the course, and the seminar participants at the Universiteit van Amsterdam. Eva Crabbé meticulously typed the entire manuscript.

## MOTIVATION

Upon returning to the USA from a sabbatical leave in several European countries a colleague was audited by the IRS (the USA tax office). To her amazement the audit contained a nice surprise, as it appeared that she had grossly understated her deduction for business expenses. Being an economics professor, she inquired about the reasons and found out the auditor had simply added all her European bills without regard for currency and exchange rates. While in some parts of Europe currency unions are actively debated, it may be still a while before we have a single world currency and the IRS can simply go ahead. At present, there are almost as many currencies as countries, and since the breakdown of the Bretton Woods arrangement the most actively internationally traded currencies experience considerable movements of their exchange rates at all frequencies.

While the disregard or ignorance of the IRS may seem a little incredible, there are a number of well known and less well known stylized facts about the empirical behavior of exchange rates that are often ignored in empirical and theoretical economics research. For example, the highly interesting target zone literature commonly employs the small scale monetary model which, just as most other reduced form structural models, has been firmly rejected as a parsimonious modeling device. This gives the theoretical predictions an unnecessary disadvantage in confrontation with the data (the theory may fail not for its essential contribution). The success story of e.g. neoclassical growth theory was made by the way it explained and integrated Kaldor's empirical regularities. The purpose of these lecture notes is therefore to collect and expose the empirical regularities which have been found in the movements of exchange rates, so as to provide a skeleton for future empirical and theoretical work .

The focus of this essay is necessarily kept quite narrow on the high frequency, nominal exchange rate behaviour of the well traded currencies. In this way we can provide an indepth statistical treatment, develop a sound economic intuition and collect new ideas for future research. Related variables like forward and futures rates, interest rates, commodity prices and asset prices will be treated when the occasion arises. The behavior of the exchange rates of minor currencies, like black market rates, receive a similar treatment. Elsewhere in this volume exchange rate models are extensively dealt with and the reader is urged not to read this as an essay on measurement without theory. The regularities of the relation between exchange rate regimes and macro variables like GDP or employment are treated in the chapter by Eichengreen. The main purpose of this essay is, nevertheless, to provide the student and researcher with a number of facts about nominal exchange rates on which future research can, and perhaps should, be based.

Before we set out, we like to note that over the years a number of high quality surveys on the topic have appeared. The interested reader is urged to consult Mussa (1979), Levich (1985), and Frankel and Meese (1987). A comprehensive account of the econometrics of exchange rates is provided by Taylor (1986), Diebold (1988), Baillie and McMahon (1989). Hodrick (1987) gives an excellent survey of the efficiency issue. De Grauwe (1989) discusses on an intuitive level exchange rate behavior from the broader macro and historical-institutional perspective. For a number of reasons, we believe, the present notes may have a positive clearing price. First, international finance is a rapidly developing field, so that a number of important new results are not adequately covered by the previous essays. Second, included in this survey are a number of statistical techniques which are essential for researchers in the area, but are not always easily accessible. Third, these notes emphasize the distributional aspects of exchange rate movements and what we can learn from this economically, which is typically not the approach taken in the economics literature.

The first section collects a number of stylized facts which are the stepping stones for exchange rate modeling. A number of these facts are singled out for an indepth treatment in section 2, i.e. the unit root property, the fat tail property and the clustering phenomenon respectively. Section 2 includes a number of technical results that are useful for the researcher.

## 1. EMPIRICAL REGULARITIES

Before we can report on these, we have to define the variables of our interest, which in turn depend on the type of questions we face. The spot foreign exchange rate e.g. seems the obvious candidate variable for trade related questions, because the exchange rate is the variable that clears the market for exports and imports. As of today, however, there are several economic arguments which suggest that for many questions the foreign exchange rate return rather than the exchange rate level is the relevant economic variable<sup>1</sup>. The major cause of short-run foreign exchange rate movements are international capital movements. For example, the net daily turnover on all foreign exchange markets of the world in April 1989 was 540 billion US dollars on average, which was 40 % more than the total mass of all foreign official reserves, and of which only 3 % was trade related. A basic presumption in finance is that investors equalize returns, corrected for uncertainty. Given the predominance of capital movements it seems therefore logical to focus on the returns rather than the levels.

---

<sup>1</sup> The return is often measured as the logarithmic difference of the level. For the relatively small day to day or week to week changes exhibited by most well traded currencies, this yields a rather good approximation to the exact definition of a return. For e.g. black market rates the logarithmic difference may not be appropriate.

There are two additional benefits from concentrating on the returns. Numeraire conventions are an important factor favoring the logarithmic transformation. The British and the continental notations are the same for the logarithm of the exchange rate, except for the sign. Hence, the sample moments are 'identical' under the two conventions. In this way, Siegel's paradox, due to Jensen's inequality, i.e.  $1/E[x] \neq E[1/x]$  in general, is circumvented. Another problem is the denomination of a currency. However, when exchange rate returns are used, one obtains a unit free measure.

The stylized facts are classified as follows. First, several facts constitute so called no (possibility of) arbitrage conditions. And consequently have direct economic content. Second, other facts are mere statistical regularities for which we currently lack a good economic explanation. A third category comprises some 'negative' results, artifacts say, i.e. regularities which are commonly hypothesized but for which not much empirical support has been found.

### 1.1. No arbitrage conditions

That returns are the variables on which we want to focus our attention for our economic investigations is corroborated by the first stylized fact.

**Fact 1 [Unit Root Property].** The logarithm of the nominal exchange rate for two freely floating currencies is non stationary, while the first difference is stationary<sup>2</sup>. □

---

<sup>2</sup> A stochastic process  $\{s(t)\}$ , where  $s(t)$  is a random variable and  $t \in \mathbb{N}$ , is said to be stationary if for any positive integer  $k$  and any points  $t_1, \dots, t_m$  the joint distribution of  $\{s(t_1), \dots, s(t_m)\}$  is the same as the joint distribution of  $\{s(t_1+k), \dots, s(t_m+k)\}$ , i.e. the joint distribution is invariant under a time shift. A process  $\{s(t)\}$  is weakly or covariance stationary if  $\text{cov}(s(m), s(k))$  depends only on the time difference  $|m-k|$ .

Let  $s(t) = \log S(t)$ , where  $S(t)$  is the spot rate and  $\log$  stands for the natural logarithm, then fact 1 can be restated as follows. The first order autoregressive stochastic process  $\{s(t)\}$

$$(1.1) \quad s(t) = \lambda s(t-1) + \epsilon(t), \quad \lambda = 1,$$

and  $\{\epsilon(t)\}$  stationary contains a unit root  $\lambda = 1$ . The knife edge value induces the non stationarity of  $\{s(t)\}$  (note :  $\{s(t)\}$  would be stationary if  $|\lambda| < 1$ ). Table 1 reports a number of test results of the  $H_0 : \lambda = 1$  for 475 Thursday closing quotations of the Canadian-U.S. dollar spot exchange rate from 1973 to 1983. Because  $s(t)$  will be non stationary under the null hypothesis, the usual critical values of the t-tests do not apply. Appropriate critical values are provided in Table 2. Both tables are taken from Hols and De Vries (1991), but are representative for the area, cf. Baillie and McMahon (1989, ch. 4).

Table 1.1 : Unit root tests\*

	$\tau$	$\tau_{\mu}$	$Z(\ell=2)$	$Z(\ell=10)$	$Z^*(\ell=2)$	$Z^*(\ell=10)$
Log-levels	0.823	-0.068	0.672	1.194	-1.005	-0.849
Returns	-12.106	-12.618	-19.201	-18.664	-19.377	-19.065

\* The test statistics are the Dickey-Fuller statistics  $\tau$ ,  $\tau_{\mu}$ , see Fuller (1976), for the models  $x_t = c + x_{t-1} + \beta(x_{t-1} - x_{t-2})$  with  $c$  zero or unrestricted, and the Phillips statistics  $Z(\ell)$ ,  $Z^*(\ell)$ , see Phillips (1987), for the model  $x_t = c + x_{t-1}$  with  $c$  zero or unrestricted and truncation lag  $\ell$ . The variable  $x_t$  refers to either the log exchange rate level or first differences.

Source : Hols and De Vries (1991).

Table 1.2 : Unit root simulations\*

Test statistic	Distribution	Probability of a smaller value	
		0.025	0.975
$\tau$	Normal	-2.26	1.66
$\tau$	Cauchy	-1.98	1.36
$\tau_{\mu}$	Normal	-3.12	0.23
$\tau_{\mu}$	Cauchy	-3.85	0.28

\* The table corresponds to Table 8.5.2 in Fuller (1976) for the normal distribution and sample size  $n = 474$ ; in addition it gives the critical values if the innovations are Cauchy distributed. The table is based on 10,000 replications, see Gielens and De Vries (1990).

Source : Hols and De Vries (1991).

While any student with some experience in applied econometrics would be cautious with reporting exactly  $\lambda = 1$  as a fact of life, economic intuition strongly favors this specific value. In efficient markets all information at time  $t-1$  is incorporated in the price  $s(t-1)$ ,  $\epsilon(t)$  captures the unanticipated elements, and hence eq. (1) is indeed a 'no arbitrage condition', see Le Roy (1989). (See section 2 for a slight modification of this statement due to the interest differential.) It has some amazing implications. For example, if we are willing to make the additional assumption, which was often made in the older literature, that the  $\epsilon(t)$  are independent and identically distributed (i.i.d.), then  $s(t)$  follows a random walk and hence  $s(t)$  eventually crosses any level  $s \in \mathbb{R}$ , with obvious ramifications for exchange rate related variables. These issues are elaborated on further in section 2.

The following two no arbitrage conditions are the centerpieces of the international money market. The highly automated information processing allows for efficient trade and arbitrage between different financial centers. Direct purchase of a particular foreign currency or indirect purchase via a third currency (financial center) should cost the same :

Fact 2 [Triangular Arbitrage Condition]. If the logarithm of two different dollar spot rates, say the DM/US rate  $s_1(t)$  and the inverse BP/US rate  $-s_2(t)$  are added, this yields the logarithmic DM/BP cross rate  $s_3(t)$  :

$$(1.2) \quad s_1(t) - s_2(t) = s_3(t). \quad \square$$

Because this equality holds very well in practice, and because some of the univariate statistical properties are common to all exchange rates, it should be the case that these statistical properties are invariant under addition, see also fact 9. This might be a very useful fact for any axiomatic approach to the distribution of exchange rates which, as of today, is non existent. Inter alia, note that eqs. like (2) in a multivariate context restrict the dimensionality of the covariance matrix of eqs. (1), i.e. implying a singular multivariate distribution.

Let  $F(t)$  be the forward foreign currency rate at time  $t$  of a forward contract with delivery date  $t+1$ ,  $I(t)$  and  $I^*(t)$  are the domestic and foreign one period nominal interest rates; and let  $C(t)$ ,  $P(t)$  denote the prices of a foreign currency call and put option with exercise price  $X$  that expires at  $t+1$ . Now, investing in a local bond, with a return of  $1 + I(t)$ , should yield the same as investing in a bond of equal quality abroad and exchanging the future proceeds at the current forward rate, i.e.  $(1 + I^*(t)) F(t)/S(t)$ . Similarly, directly buying a forward contract for future exchange against rate  $F(t)$ , should cost the same as taking an indirect hedge through buying a call selling (writing) a put, i.e. a so called "reversal", and bringing forward the cost of borrowing the difference  $C(t) - P(t)$ . To see that this reversal duplicates a forward contract, note that the trader using the options market gains (loses) dollar-for-dollar by the amount the future spot rate is above (below) the exercise price  $X$ ; similarly a trader using the forward market gains (loses) the difference between the future spot rate and the forward rate.

Fact 3 [Parity Conditions]. The following relations, covered interest rate parity :

$$(1.3) \quad \frac{F(t)}{S(t)} = \frac{1 + I(t)}{1 + I^*(t)},$$

and put-call parity :

$$(1.4) \quad F(t) = X + [C(t) - P(t)][1 + I(t)],$$

hold for all major traded currencies. □

The covered interest rate parity condition is often stated in the following approximate format

$$(1.5) \quad f(t) - s(t) = I(t) - I^*(t),$$

where  $f(t) = \log F(t)$ . Discrepancies in these relations arise due to transactions costs, bid-ask spreads and capital controls, see e.g. Levich (1985), and Baillie and McMahon (1989, ch. 5). Usually some wedge between the left hand side and right hand side of eqs. (3) and (4) exists, suggesting arbitrage opportunities. Most of the time, however, transactions costs, albeit small, prevent a profitable round-trip. While (3) usually holds up very well if offshore (Euromarket) interest rates are used, this is not the case for onshore interest rates. The discrepancy is mostly due to the existence of capital controls. During times of strains within e.g. the EMS, the disparity usually increases due to the risk of a realignment, which renders the forward market thin. For futures contracts a condition similar to (3) holds.

Evidently, eqs. (3) and (4) can be combined to yield an arbitrage relation between interest rates and currency options. An interesting 'new' fact arises from combining facts 1 and 3. To introduce this new fact we need a new concept. Recall the definition of stationarity in fn. 2, and the fact that while  $s(t)$  is found non stationary the return  $s(t) - s(t-1)$  is stationary. This univariate differencing to obtain a

stationary series can be generalized to a multivariate setting. Two non-stationary random variables, say  $s(t)$  and  $f(t)$ , are said to be cointegrated if some linear combination  $x(t) = s(t) + af(t)$ , say, is stationary and where  $a$  is said to be the cointegrating 'vector' (in the univariate case one could say  $s(t)$  and  $s(t-1)$  are cointegrated if one replaces  $f(t)$  by  $s(t)$  and sets  $a = -1$ ). If there does not exist such a linear combination, the two variables are not cointegrated. Now suppose that the interest differential on the right hand side of eq. (5) is a stationary random variable and recalling fact 1, then implies:

**Fact 4 [Cointegration].** The spot rate  $s(t)$  and the accompanying forward rate  $f(t)$  are cointegrated with cointegrating vector  $a = -1$ , while different (freely floating) spot rates are typically not cointegrated. □

If  $s(t)$  and  $f(t)$  are cointegrated, then  $f(t)$  and  $s(t+k)$  will be cointegrated as well. To see this, suppose  $f(t)$  and  $s(t+k)$  were not cointegrated, then  $s(t+k) - f(t)$  would be non stationary, and hence in combination with fact 1, it follows that both  $s(t+k)$  and  $f(t)$  could wander infinitely far away from each other. This implies infinitely high risk premia, defying the existence of a forward market (a direct analytical proof is to add the stationary increment  $s(t+k) - s(t)$  to the difference  $s(t) - f(t)$ ). Thus  $s(t+k) - f(t)$  being stationary makes sense. Hence, Granger (1986) concluded that in an efficient market contracts which are related to the same asset should be cointegrated. Evidence of this cointegration relation is reported in Table 3. The table reports OLS estimates for the equation

$$s(t+k) = a + b f(t) + \epsilon(t+k),$$

with  $k$  equal to the number of trading days during a thirty day forward contract. The table is based on Baillie and McMahon (1989, ch. 4), who also test against nonstationarity of the residuals. Except for the Dollar-Yen rate, the results are convincing. See Hakkio and Rush (1989) for additional evidence.

Table 1.3 : Cointegration between  $s(t+k)$  and  $f(t)$ \*

Country	$\hat{a}$	$\hat{b}$
U.K.	-0.0187	1.0135
West Germany	-0.0301	0.9802
France	-0.0379	0.9852
Italy	-0.0892	0.9886
Switzerland	-0.0298	0.9756
Japan	-0.8347	0.8476
Canada	-0.0095	0.9599

\* Estimates are based on a sample of U.S. dollar daily foreign exchange quotations in New York over the period 1980-1985.

Source : Baillie and McMahon (1989, ch. 4).

We started the discussion of fact 4 by assuming that the interest differential in eq. (5) is stationary, and in combination with the non-stationarity of  $s(t)$  we found  $f(t)$  has to be cointegrated with  $s(t)$ . Vice versa, given that  $f(t)$  and  $s(t)$  are cointegrated, and if nominal interest rates are non stationary, then  $I(t)$  and  $I^*(t)$  are cointegrated as well. Thus cointegration between one set of variables induces important stochastic restrictions on other sets of variables. The non cointegrating feature of different (freely floating) spot rates is also important, as it yields indirect support for the efficient market hypothesis discussed in the next section. On the other hand, this observation does not apply to cross rates, i.e. recall eq. (2). Moreover, it does not apply to different (cross) rates from currency blocs like the EMS.

Related to the spot and forward rate movements is the following fact on the relative importance of the innovations.

**Fact 5 [News Dominance].** The variation in the spot returns  $s(t+1) - s(t)$  is much larger than the variation in the forward premium  $f(t) - s(t)$ , and ipso facto the interest differential.  $\square$

If the realized spot return is decomposed into an anticipated and unanticipated or news part, and if we identify the anticipated part with the forward premium, then fact 5 says that the news factor dominates. That  $|f(t) - s(t)|$  is small relative to  $|s(t+1) - s(t)|$  is not too surprising given the way the forward market operates. Banks which provide forward contracts hardly take any open positions, but instead try to reverse their position by an opposite contract. Thus banks basically perform a clearing or matching function and hence the risk premium can be relatively small. The interest rate parity condition (5) implies that the same conclusion applies to the relative variability of the spot returns vis-à-vis the interest differential. We also note that the interest differential, and hence the forward premium, are usually autocorrelated.

**Fact 6 [Calendar Effects].** There are significant time of trade effects, such as the day of the week, on the location and scale of the process.  $\square$

In particular, positive Wednesday and negative Thursday dummies for the mean, and positive Monday dummies for the variance are found in the data, see e.g. Taylor (1986) and Baillie and McMahon (1989). These effects are often due to institutional factors. For example the opposite Wednesday-Thursday effects on the mean are caused by different delays in settlement for dollar and non dollar contracts, and the positive Monday effect on the scale arises from uncertainty induced by market closure over the weekend. The institutional set-up of the currency market also explains why psychological barriers, i.e. less trades take place in the neighborhood of rounded numbers such as DM/US = 2.00, seem to exist in dollar rates, while the inverse rates do not exhibit this pattern. The reason is that all quotations on the Reuter's screens are given on a per dollar basis. This, though, does of course not explain the existence of such psychological barriers in itself, see De Grauwe and Decupere (1991). In the context of security prices it has been observed that rounding effects may seriously bias

estimates of the moments. Care has to be taken when multiple time series are investigated, such as the forward rate and the related future spot rate, that the series are appropriately matched. Typically high frequency data are not equally spaced in time, and this may affect the results. Also, there may be simply too many data to check the recording consistency by hand, and hence appropriate filters may have to be employed. Wasserfallen (1989) and Goodhart and Figliuoli (1991) discuss the properties of data recorded at the highest possible frequency.

## 1.2. Statistical regularities

We turn our attention to regularities which have a sound statistical basis, but for which no convincing economic explanation has been established. On first sight the unit root scheme (1) leaves disappointingly little room for further investigations, because no other variables than the lagged rate appear on the right hand side. As it turns out, though, a lot more can be said about the stationary innovations  $\epsilon(t)$ . The evidence is classified according to the features of the unconditional and the conditional distribution, and we start with the former.

**Fact 7 [Fat Tail Phenomenon].** Exchange rate returns, irrespective of the regime, when standardized by their scale, exhibit more probability mass in the tails than distributions like the standard normal distribution. □

Loosely speaking this means that extremely high and low realizations occur more frequently than under the hypothesis of normality. Ipso facto one has to exercise care in removing so called outliers so as not to reject the good with the bad. A related fact is that the density of the returns is more peaked than the normal density. A popular measure for this latter fact is the kurtosis, but note that a positive kurtosis does not necessarily indicate the fat tail phenomenon as is

sometimes supposed (see section 2). The distinction between thin tailed distributions like the normal distribution and fat tailed distributions is that the former have tails which decline exponentially fast, while the latter distributions have tails which decline by a power. A simple condition, known as a regular variation at infinity, operationalizes the fat tail property. Let  $F(t)$  be a distribution function, then if

$$(1.6) \quad \lim_{t \rightarrow \infty} \frac{1-F(tx)}{1-F(t)} = x^{-\alpha}, \quad \alpha > 0,$$

holds for some  $\alpha$  and positive  $x$ , then  $F(t)$  is said to be regularly varying with tail index  $\alpha$ . Loosely speaking,  $\alpha$  can be identified with the number of moments that exist (in case of the Student-t distribution  $\alpha$  equals the degrees of freedom), and thus represents a measure of tail fatness. Nonparametric estimates of  $\alpha$  for three different periods are recorded in Table 4, which is based on Koedijk, Stork and De Vries (1992). Parametric estimates reported in Westerfield (1977) and Boothe and Glassman (1987), reveal the same message but are hampered by the non-nestedness of the different parametric models. This is not the case for the non-parametric approach.

Table 1.4 : Tail Indexes \*

Parameter	$\alpha$		
	fix(62-71)	float(73-84)	float(73-91)
# of observations	485	605	962
Deutsche Mark	1.20 (0.86,1.52)	3.45 (2.53,4.37)	3.51 (2.75,4.28)
Pound	1.14 (0.82,1.45)	3.21 (2.35,4.06)	3.58 (2.80,4.36)
Yen	1.26 (0.91-1.60)	2.74 (2.01-3.47)	2.74 (2.15-3.34)
Guilder	2.42 (1.75,3.08)	3.35 (2.45,4.24)	3.45 (2.70,4.21)
Canadian Dollar	1.59 (1.15-2.03)	2.66 (1.95-3.37)	2.99 (2.34-3.64)

\* Estimates are based on weekly return data of US dollar exchange rates. Method of estimation is the Hill estimator, see section 2, and asymptotic standard errors are recorded in brackets.

Source : Koedijk, Stork, and De Vries (1992).

From the table it is apparent that exchange rate returns are heavily fat tailed, and the more so the more they are regulated. The economic intuition behind this fact is an odd basket of arguments, some of which may have to be thrown out on second thoughts. For example, the overshooting property maintains that as floating rates carry the burden of adjustment in the presence of sticky commodity prices and wages, exchange rates tend to overshoot. Also, some of the other properties presented below, i.e. additivity and volatility clustering, are connected with the fat tail property. In general, though, one finds that the more a rate is left to float freely, the thinner the tails, see Table 4 where the  $\alpha$ 's for the float are significantly higher than the  $\alpha$ 's for the period of almost fixed exchange rates.

This corroborates the Friedman presumption that the free float produces a smoother adjustment than the other regimes.

The second statistical fact relates to the third central moment of the unconditional distribution.

Fact 8 [Skewness]. Exchange rate returns of currencies which experience similar monetary policies exhibit no significant skewness, while dissimilar policies tend to generate skewness. □

Skewness appears in the data if an exchange rate predominantly drifts one way or another. This is often caused by a disparity between monetary policies, like a hyperinflation versus a deflationary policy. Less extremely, within the European Monetary System (EMS) the weaker currencies were repeatedly devalued vis-à-vis the stronger currencies, because of the devaluation bias inherent to a system of semi fixed currencies. The following fact is somewhat more surprising.

Fact 9 [Additivity]. The distribution of the largest returns when aggregated over time or across exchange rates is invariant up to a location and scale adjustment. □

The precise meaning of this statement will only become clear from the concepts introduced in section 2. The additivity property, in combination with the existence of all moments, is the defining characteristic of the normal distribution. Mandelbrot (1963a,b) first observed that the property was also present in non-normal fat tailed distributed return series. The additivity property across different exchange rates follows almost directly from fact 2, the triangular arbitrage condition, and fact 7, the fat tail property. Because, if two (independent) random variables have distributions which are regularly varying, i.e. satisfy (6), then the distribution of the sum is regularly varying as well (see section 2).

The conditional distribution, i.e. the distribution of  $\epsilon(t)$  in eq. (1) given the observed history  $\{\epsilon(t-1), \dots, \epsilon(t-n)\}$ , is dominated by the following fact.

**Fact 10 [Volatility Clusters].** Periods of quiescence and turbulence tend to cluster together.  $\square$

Again, this fact was already observed by Mandelbrot, but was more or less neglected until recently. Return series were often subjected to tests of serial dependency, but such tests focussed primarily on the autocorrelation properties in the mean or location of the process and relied on the popular ARMA representation of time series. However, not much of such dependency could be detected. The clusters of volatility regularity suggests, instead, that autocorrelation in the scale of the process  $\{\epsilon(t)\}$  is the more typical feature<sup>3</sup>. The convenient GARCH scheme developed at the beginning of the eighties was instrumental in popularizing this fact in economic modeling. By letting the conditional variance depend on the past squared innovations, it directly captures the effect that once the market is heavily volatile, it is more likely to remain so than to calm down, and vice versa.

Formally, the GARCH (1,1) model reads as follows, see also section 2,

$$(1.7) \quad \epsilon(t) = X(t) H(t)^{1/2}$$

$$(1.8) \quad H(t) = \omega + \lambda \epsilon(t-1)^2 + \beta H(t-1),$$

and where  $X(t)$  are i.i.d. innovations. Some typical parameter estimates for the case  $X(t)$  is Student-t distributed with  $v$  degrees of freedom are reported in Table 5.

---

<sup>3</sup> We may want to deliberately avoid to use the concept of variance because the fat tail property may imply that the second moment is not defined, while other measures of scale like the interquartile range always exist.

Table 1.5 : The GARCH model\*

exchange rate	parameter		
	$\lambda$	$\beta$	$v$
French Franc/US Dollar	0.15	0.79	6.56
Yen/US Dollar	0.08	0.90	5.91
Deutsche Mark/US Dollar	0.11	0.86	14.93
Belgium Franc/French Franc	0.71	0.21	2.46
Belgium Franc/Yen	0.17	0.63	4.44
Belgium Franc/Deutsche Mark	0.58	0.39	3.06

\* The results for the first three exchange rates are taken from Baillie and McMahon (1989, ch. 4) and are based on a sample of 1200 daily observations, while the latter three are based on 500 weekly observations reported by De Ceuster (1992). Details of the estimation are given in these studies.

The GARCH parameters are significantly different from zero in all cases, and hence volatility clusters are clearly present. Also, as will become evident from the discussion in section 2, the parameter estimates of  $(\lambda, \beta, v)$  corroborate the fat tail phenomenon of fact 7, and are in line with the results of Table 4. Specifically, the intra EMS rates again display fatter tails. Hence, the explanation for fact 7 may just be the volatility cluster effect. Unfortunately, this shifts the problem towards explaining fact 10, because the economics behind this latter fact are not well understood as of yet. Note, however, that taken together the absence of dependency in the mean and positive autocorrelation in the scale of the process is not inconsistent with risk neutral agents arbitraging in the levels of the returns<sup>4</sup>. The converse, i.e. fact 10 implicitly rejecting risk aversion is not necessarily true, though (see e.g. LeRoy, 1973).

<sup>4</sup> It has also been found that the volatility gets transmitted from one market to another market where the same exchange rate is quoted at different times. In this way uncertainty concerning money market announcements spreads around the world.

General specification tests, which were derived as a byproduct from chaos theory, have confirmed that nonlinearities in the data generating process are clearly present. But as of today this has not led to serious amendments on the ARCH model. The well known deterministic nonlinear chaos models have not made much inroads, because their deterministic features and data requirements for falsification render it unsuitable for economic analysis.

### 1.3. Artifacts

There is a number of relationships which make sense on the basis of economic principles, but for which the empirical evidence is only marginal. Some of these relations are nevertheless frequently hypothesized in theoretical work, because they are so 'convenient', or because they are part and parcel of current paradigms. Needless to say that empirical work which basis itself on such a theoretical exposé often fails because one of the maintained hypotheses<sup>e</sup> is grossly at variance with the data (this is, of course, not a necessity). In this subsection we collect a number of these 'artifacts'. We remind the reader that our focus is on the high frequency behavior of freely traded currencies, and that the artificats may become facts in a different context (e.g. PPP fails on high frequency data, while it cannot be rejected on the very low frequency data).

One form of the efficient market hypothesis, i.e. when the market uses all relevant information and uses this information correctly to determine exchange rates, in conjunction with risk neutrality implies that the forward rate is an unbiased predictor of the future spot rate :

$$(1.9) \quad f(t) = E[s(t+1)].$$

where  $E[.]$  is the expectations operator given the information set at time  $t$ . Combining eqs. (5) and (9) then yields the uncovered interest rate parity condition.

Fact 11 [Uncovered Interest Rate Disparity]. This condition

$$(1.10) \quad E[s(t+1)] - s(t) = I(t) - I^*(t),$$

is usually rejected by the empirical material.  $\square$

Tests of eq. (10) are marred by the overlapping data problem, see Hansen and Hodrick (1980), conditional heteroskedasticity, see Hodrick (1987, ch. 3), and cointegration, see Hakkio and Rush (1989). Nevertheless, the unbiasedness hypothesis has been rejected time and time again, see Baillie and McMahon (1989, ch. 6), Hodrick (1987, ch. 3) and Fama (1984). This is not necessarily evidence against market efficiency, only against the particular model of market equilibrium on which the tests are based. In particular, the unbiasedness hypothesis (9), almost always, presupposes risk neutral agents. Therefore research has turned to testing efficient market models which generate a nonzero risk premium, such as the consumption based CAPM. To this end the ARCH type error structure is employed because its conditional heteroskedasticity conveniently captures the idea of a time varying risk premium. As of to date, however, this research is largely inconclusive, see Frankel and Meese (1987). Other explanations are based on market inefficiency, expectational failures and non ergodicity of the data due to Peso problems. To conclude, the hunt for a plausible econometric specification generating a risk premium that explains the failures of (9) or (10) is still on.

From the trade balance point of view one would expect an intimate relation between relative prices and exchange rates. Let  $p(t)$  and  $p^*(t)$  denote the logarithm of the domestic and foreign price levels. Purchasing power parity (PPP) is said to prevail in absolute terms if

$$(1.11) \quad s(t) = p(t) - p^*(t).$$

and in relative terms if

$$(1.12) \quad \Delta s(t) = \Delta p(t) - \Delta p^*(t),$$

where  $\Delta$  is the difference operator.

Fact 12 [No PPP]. Neither form of PPP holds in the short run, while there is some evidence favoring (relative) PPP in the long run.  $\square$  The absence of PPP in the short run follows from the fact that aggregate price levels or indexes are relatively sticky in the short run, due to e.g. the periodic fixing of wage contracts. This, in combination with eq. (1), renders the failure of (11) or (12) as a small surprise. In other words, the real exchange rate  $q(t)$ , where

$$(1.13) \quad q(t) = s(t) + p^*(t) - p(t),$$

is indistinguishable from a unit root process in the short run. But persistent deviations have been observed over much longer horizons than, say, a year. Only over time horizons of e.g. a century have terms of trade effects, caused by e.g. relative productivity changes, been detected, see Frankel and Meese. Also, currencies which experience a hyperinflation vis-à-vis stable currencies usually have depreciating exchange rates, corroborating fact 8. But again, in general detection of PPP is deterred by statistical features of the data, like unit roots and cointegration, and these have only recently been tackled head on.

Fixed and semi-fixed regimes exhibit a number of interesting idiosyncrasies. A celebrated relationship exists between the trade balance  $B(t)$  and the logarithmic (real) exchange rate. Disaggregating  $B(t)$  into domestic and foreign demand and supply, rewriting this equation into elasticity format and differentiating with respect to  $q(t)$  yields a "positive" effect of a devaluation on the trade balance if the Marshall-Lerner condition is satisfied, i.e. the sum of the absolute demand elasticities must exceed 1. Received wisdom has it that the elasticity condition holds, albeit not in the short run due to price

rigidities which produce an initial deterioration of  $B(t)$ , i.e. the typical J-curve effect. Recent econometric investigations which directly evaluate the connection between  $q(t)$  and  $B(t)$ , instead of the indirect evidence produced by estimating trade elasticities, however, do not find any definite relationship, see Rose (1991). Given the absence of PPP stated in fact 12, this is not too surprising. A relatively new phenomenon is the S-shaped behavior of exchange rates within target zone. This is extensively discussed in the chapter by Bertola.

A typical aspect of pegged exchange rates is the  $n$ -th currency problem. Because  $n$  currencies only generate  $n-1$  exchange rates relative to a numeraire currency (and all other cross rates follow from the triangular arbitrage condition (2)), this leaves one degree of freedom: with  $n-1$  relative prices, the level of the  $n$ -th currency stock can be chosen freely. This turned out to be the case under the Bretton-Woods agreement, whereby the United States took its liberties, until the other countries were no longer willing to swallow the increase in dollars. A similar degree of freedom exists within the EMS. And one of the questions is whether Germany plays the  $n$ -th country role, i.e. the so called German dominance hypothesis. This is briefly discussed in section 2.

The pressure on fixed or managed exchange rates which builds up due e.g. diverging inflation rates is often countered through official interventions. One could say that the foreign exchange market is an asset market with sanctioned insider trading. Nevertheless, the intentions of the central banks are often revealed indirectly through their publicly announced targets concerning other variables like the interest rate. While unsterilized intervention may be effective because it changes the money supply, the effectiveness of sterilized intervention hinges on the non-substitutability of foreign and domestic assets.

Fact 13 [Ineffectiveness of Sterilized Intervention]. Most evidence shows that sterilized intervention has no or only temporary effects on the exchange rate. □

Alternative means for managing exchange rates are capital controls. A special case of this is the use of dual exchange rates. Under a dual exchange rate regime, different parts of the balance of payments are cleared against different rates. This, of course, induces the possibilities for (illegal) arbitrage. If exchange controls drive too big a wedge between the official rate and the shadow free market rate, the latter comes into the open in the form of a black market rate. Often such a market is unofficially tolerated, to take away the greatest strains from the system. Another arbitrage scheme is currency substitution which occurs when some of the roles of the local currency are partly taken over by a foreign currency<sup>5</sup>. Indirect currency substitution is said to occur when other foreign assets are being substituted for other domestic assets, like in the case of bond substitution.

Fact 14 [Inelastic Currency Substitution]. The elasticity of direct currency substitution is not very high. □

Habit formation, legal restrictions, and the fact that the rate of return on money is dominated by other assets, severely limit the possibilities and rationale for direct currency substitution (see De Vries, 1988, for estimates of the elasticity of substitution). It must be said though that in countries where one would a priori expect a high elasticity, such as in the case of a hyperinflation, a lack of good data material has prevented reliable measurement of the elasticity of currency substitution. Evidently, within one jurisdiction, the elasticity of substitution between coins, paper money and plastic money is very high. Nevertheless, there is evidence that even during

---

<sup>5</sup> During the Israeli hyperinflation it became illegal to transact in dollars, but nevertheless the dollar functioned as a unit of account in e.g. housing contracts. In Mexico the dollar has functioned as a means of payment, and Panama has no currency of its own but uses the US dollar.

hyperinflations substitute monies are not used on a large scale, see Barro (1972).

Considerable attention has been devoted to the impact of (conditional) exchange rate variability on the volume of international trade. This activity notwithstanding, we have the following conclusion.

**Fact 15 [No Volatility Impact on Trade].** There does not appear to exist an unambiguous relationship between (conditional) exchange rate (return) volatility and international trade. □

The failure to turn up the presumed negative relationship is due to several factors. Theoretical models that incorporate the possibilities for hedging and employ a general equilibrium setting, do not necessarily imply that exchange rate volatility is detrimental to trade, see Viaene and De Vries (1992). The measurement of the volatility effects, moreover, is not an easy job. For example, inclusion of a volatility measure in a regression purporting to explain trade flows may be marred by the constructed regressor problem. The measurement of the conditional volatility could conceivably be improved by exploiting the fact of volatility clustering through an ARCH type representation. On the other hand, longer term volatility as signified by the sustained increases and decreases in the value of the US dollar during the eighties has left its imprints on trade. Goldstein and Kahn (1985) provide a survey of the other trade, price and exchange rate issues.

The relation between the exchange rates and other macroeconomic variables in general, except those which appear in the no arbitrage conditions, can be succinctly worded as follows:

**Fact 16 [No Fundamentals].** The predictions from (high frequency) reduced form exchange rate models do not outperform simple no change forecasts. □

Note that this fact is in conformity with the unit root property stated in fact 1. While we will see that eq. (1) is just a simple no arbitrage condition, it took a long time before it was put to test.

given the economist's focus on structural models, and its full implications are yet being swallowed. The most damaging evidence against the fundamentals approach was delivered by Meese and Rogoff (1983). Meese and Rogoff compared out of sample forecasts of reduced form structural models (using actual realized values of the explanatory variables) with the no change forecast of a random walk. Table 6 summarizes some of their results. The absence of a fundamentals model also impairs the recently popular tests for excess exchange rate volatility based on variance bounds or bubbles, because these are all conditional on using the correct fundamentals model. Not knowing this model renders such tests virtually inapplicable. The macro oriented exchange rate literature after the demise of Bretton Woods has largely been an epitaph on the fundamentals models of exchange rates. This has nevertheless been a positive process, because it stimulated the inquiry into the behavior of  $\epsilon(t)$ , generated numerous of the facts recounted above, and it has been useful for economic modelling as is evidenced by the other chapters in this volume.

Table 1.6 : Root mean square forecast errors\*

Exchange Rate	Forecast Horizon	Random Walk	Monetary Model
US Dollar/Deutsche Mark	1 month	3.72	3.17
	6 months	8.71	9.64
US Dollar/Yen	1 month	3.68	4.11
	6 months	11.58	13.38
US Dollar/Pound	1 month	2.56	2.82
	6 months	6.45	8.90

\* Exchange rates are in logarithms, hence the forecast error is approximately in percentage terms. The monetary model derives from the logarithmic difference of the domestic quantity equations and the logarithmic PPP relation, see section 2.1.

Source: Meese and Rogoff (1983, Table 1).

## 2. THEORY

### 2.1. Arbitrage and unit roots

In this section we single out the three dominant statistical issues, i.e. nonstationarity, fat tails and volatility clusters, for further investigation. This is not to say that the other facts are of lesser importance, but these are extensively treated elsewhere. Turning to the topic of this subsection, we like to remind the reader of facts 1 (unit roots), 16 (no fundamentals), and 4 (cointegration). We will first argue why economists have not been able to develop a convincing model of high frequency exchange rate behavior on basis of economic fundamentals. Fortunately, this does not imply that economics has nothing to say. In fact, consistent with most economic theories, arbitrage arguments strongly suggest that we should not be able to find the stone of economic wisdom for predicting exchange rate levels. Instead, economic theory does suggest something about the way returns behave and vice versa.

To see the no arbitrage argument, recall the fact recounted above that almost no exchange in the foreign exchange market is trade related but rather most transactions <sup>are</sup> investment motivated. Now contemplate the following experiment. <sup>are</sup> Suppose one is teaching a class and offers to sell the contents of one's wallet through an English auction such as is used in selling antiques, without revealing the actual contents beforehand. Two students, however, are granted the right to see the true contents before the auction. When played in practice, one usually finds the two informed students bidding against each other, while the uninformed hardly participate. When the uninformed students are asked to guess the true contents after the bidding has ceased, most students call the winning bid, as they realize that the two informed have an incentive to outbid each other until the true value of the contents is reached. Thus all information gets reflected in

the price and the market is said to be efficient. Similarly, a known or expected exchange rate revaluation (devaluation) leads to an almost instantaneous decrease (increase) in the spot rate by the arbitrage process outlined above. Usually this rapid adjustment process is omitted from the analysis. What one is left with is the no arbitrage condition

$$(2.1) \quad s(t+1) - s(t) = \epsilon(t+1) , \quad E[\epsilon(t+1)] = 0 ,$$

i.e. all what can be said about the future spot rate(s) is contained in the current rate :

$$(2.2) \quad E[s(t+k)|s(t)] = s(t) \quad \text{for any } k > 0.$$

If we add the restriction that  $E[|s(t)|] < \infty$ , then the stochastic process  $\{s(t)\}$  is said to be a martingale<sup>6</sup>. A stronger assumption is to maintain that the  $\epsilon(t)$  are i.i.d., which renders the random walk. Because of the volatility clusters (fact 10), the random walk is unnecessarily restrictive and hence we concentrate on the unit root property<sup>7</sup>.

An important implication of the no arbitrage argument is the impossibility of trading rules. It is important because many economists and technical analysts usually have a hard time to swallow this feature

---

<sup>6</sup> Note that if e.g. the  $\epsilon(t)$  are independent,  $E[\epsilon(t)] = 0$  and  $E[|\epsilon(t)|] < \infty$  for all  $t$ , it follows that the restriction is satisfied because  $E[|s(t)|] \leq E[|\epsilon(1)|] + \dots + E[|\epsilon(t)|] < \infty$ .

<sup>7</sup> The random walk model, however, is useful to obtain intuition about the implications of a unit root. If  $\{s(t)\}$  is a random walk, then  $s(t)$  returns to  $s(0)$  infinitely often, but the expected waiting time for a return is infinite. The persistence of a random walk is also evident from the U-shaped distribution of sojourn times: the percentage of time  $\alpha$  that  $s(t) > 0$  is distributed as  $2 \arcsin \sqrt{\alpha/\pi}$ . Another interesting feature is that while two independent random walks meet infinitely often with probability one, this probability is less than one for three or more random walks. The interested reader is advised to consult the lucid elementary treatment of Feller (1970).

of efficient markets. To develop the argument suppose  $\{s(t)\}$  is a martingale with  $s(0) = \epsilon(0) = 0$ . From (2.1) we have that

$$(2.3) \quad s(t) = \epsilon(1) + \dots + \epsilon(t).$$

Because of eq. (3), we may replace the conditioning variables in (2) by

$$E[s(t+1)|\epsilon(1), \dots, \epsilon(t)] = s(t).$$

In this spirit the more general definition of a martingale allows the conditioning variables to be any stochastic process  $\{y(t)\}$  such that

$$(2.4) \quad E[s(t+1)|y(1), \dots, y(t)] = s(t).$$

Often  $\{y(t)\} = \{\epsilon(t)\}$ , but we may want to enlarge the information set by other random variables from the past. By the law of iterated expectations<sup>8</sup>

$$\begin{aligned} E[s(t+1)|y(1), \dots, y(t-1)] &= \\ E\left[E[s(t+1)|y(1), \dots, y(t)]|y(1), \dots, y(t-1)\right] &= \\ E[s(t)|y(1), \dots, y(t-1)] &= s(t-1). \end{aligned}$$

By induction we get

$$E[s(t+1)|y(1), \dots, y(k)] = s(k), \quad k = 1, \dots, t.$$

It follows that any subsequence, e.g.  $\{s(2t)\}$ , follows a martingale as well.

---

<sup>8</sup> Recall  $E[E[Y|X]] = \int \int y f_{y|x}(y|x) f_x(x) dy dx = \int \int y f_{y,x}(y,x) dy dx = E[Y]$ .

This last observation can be used to show the impossibility of trading systems. Let  $\{s(t)\}$  again be a martingale with respect to  $\{y(t)\}$ , see (4). Let  $\chi(t) = \chi(y(1), \dots, y(t))$  be a function of all past information which takes on the value 1 or 0. The value 1 is associated with playing, say investing one dollar in foreign currency with a return  $s(t) - s(t-1)$ , and 0 denotes abstention, i.e. skipping the possibility of investment. When  $\chi(n) = 1$ , the gain at the  $n$ -th trial is

$$s(n) - s(n-1),$$

and zero otherwise. The accumulated gain  $a(n)$  at time  $n$  is

$$a(n) = a(n-1) + \chi(n-1)\{s(n) - s(n-1)\}.$$

Because  $E[a(1)] = \chi(1) E[s(1)] = 0$  by definition, and using an induction argument, the unconditional expectation  $E[a(n)]$  clearly exists, i.e. it is zero. Hence the conditional expectations can be calculated as well. In particular

$$(2.5) \quad E[a(n)|y(1), \dots, y(n-1)] =$$

$$a(n-1) + \chi(n-1) \{E[s(n)|y(1), \dots, y(n-1)] - s(n-1)\} = a(n-1).$$

Thus  $\{a(n)\}$  is a martingale, c.f. eq. (4). We have proved:

**Theorem 2.1. Impossibility of Trading Systems.** Every sequence of zero-one decision functions  $\chi(t)$  changes the martingale  $\{s(t)\}$  into a new martingale  $\{a(t)\}$ . □

As a special case consider the option to halt playing altogether. In this case the decision function  $\chi(t)$  becomes  $\chi(t < t_0) = 1$  and  $\chi(t \geq t_0) = 0$  for some  $t_0$ . The function tells when to stop investing. By the above theorem it is immediate that :

**Corollary 2.1.** 'Optimal' stopping does not affect the martingale property of  $\{a(n)\}$ . □

The theorem and corollary dispel the possibility to devise profitable trading schemes (which are linear in the outcome) such as proposed by technical analysis. But if there exists structure in the higher moments of  $\epsilon(t)$ , then there may exist profitable trading rules (that are nonlinear in the outcome). Thus a risk averse agent might be able to exploit a scheme like ARCH (see below).

How to reconcile the simple scheme in (1) and (2) with the elaborate fundamentals models that are so common in economics? Consider, e.g. the simple monetary model. From the Keynesian money demand or logarithmic quantity equation we have

$$(2.6) \quad m(t) = p(t) + \phi y(t) - \Psi I(t),$$

where  $y(t)$  is logarithmic income,  $m(t)$  is the logarithm of the money stock, and  $\Psi$  is the interest semi-elasticity of money demand. Equate money demand with money supply and subtract a similar relation (with identical parameters) for the foreign country, this yields

$$(2.7) \quad m(t) - m^*(t) = p(t) - p^*(t) + \phi(y(t) - y^*(t)) - \Psi(I(t) - I^*(t)).$$

Sinning against facts 12 and 11 for the sake of the presentation, invoke PPP

$$(2.8) \quad s(t) = p(t) - p^*(t)$$

and uncovered interest rate parity

$$(2.9) \quad I(t) - I^*(t) = E_t[s(t+1)] - s(t).$$

Solve for the exchange rate from equations (7)-(9) :

$$(2.10) \quad s(t) = \frac{\Psi}{1+\Psi} E_t[s(t+1)] + \frac{1}{1+\Psi} \{m(t) - m^*(t) - \phi[y(t) - y^*(t)]\}.$$

For clarity of exposition we restate this equation as

$$(2.11) \quad s(t) = \lambda E_t[s(t+1)] + x(t) \quad 0 < \lambda < 1,$$

Through recursive forward substitution the particular (no bubble) solution to eq. (11) reads

$$(2.12) \quad s(t) = \sum_{i=0}^{\infty} \lambda^i E_t[x(t+i)].$$

If we are willing to make the assumption that the fundamentals' process  $\{x(t)\}$  is a martingale, then the no bubbles forward solution to eq. (11) implies the martingale model (1) and (2). Thus the fundamentalist view is not contradictory with the no arbitrage unit root property. The reason is that rational expectations rule out arbitrage possibilities in the forward looking model (3). Crucial for this result is that the fundamentals are a martingale. This may be more or less plausible for the high frequency returns. Typically the fundamentals, like income, display a high persistence and cannot be observed as frequently as the returns. The no change view of the fundamentals may therefore be not a bad assumption. (The fundamentals which are regularly observed, such as the interest rates, usually display the martingale property as well.) This would agree with fact 16 and the tests conducted by Meese and Rogoff (1983a). When Meese and Rogoff first published their results, see Table 6 above, these met with incredulity, and many researchers have since then tried to beat the martingale model, without much success. Nowadays the nature of the forward solution to eq. (3) is better understood.

After the demise of the structural models, economists turned to the theory of finance and embarked on large scale testing of the (weak form) efficient market hypothesis for the foreign exchange market. The absence of fundamentals on the right hand side of eq. (1.1) does not necessarily imply that the foreign exchange market is efficient. As a simple counterexample consider the stationary process which is open to arbitrage:

$$(2.13) \quad s(t+1) = \beta s(t) + \epsilon(t+1) \quad |\beta| < 1.$$

The test of market efficiency then boils down to a test for the unit root  $\beta = 1$ . While estimation of the two simple alternatives (1) and (13) can proceed by OLS, testing for  $H_0 : \beta = 1$  against  $H_1 : |\beta| < 1$  is not so simple. The reason is that if the process  $\{s(t)\}$  in (1) has been initiated in the indefinite past, then  $\text{Var}[s(t)] = \infty$  (even if  $\text{Var}[\epsilon(t)]$  is bounded and nonzero). This invalidates the asymptotic normality of  $\hat{\beta}$  which obtains if  $|\beta| < 1$ , and hence impairs the conventional t-test. Nevertheless, White (1958) obtained the limiting distribution of  $\hat{\beta} - \beta$  appropriately normalized for the cases  $\beta = 1$  and  $\beta > 1$ . On the basis of this Dickey and Fuller constructed the critical values for the t-test  $(\hat{\beta} - 1)/\hat{s}_\beta$ , see Fuller (1976, Table 8.5.2). For example, for the one sided test of  $H_0$  against  $H_1$  with 100 observations the critical value is -1.95 at the 5 per cent significance (cf. the critical value of the usual t-test is -1.65). In conducting this test different critical values apply if a constant or time trend is included in the regressions. The test can also be used in case the innovations  $\epsilon(t)$  are fat tailed (the critical values differ slightly), recall Table 1.2, or in case there exists some serial correlation (the augmented Dickey Fuller procedure).

An awkward property of the above test procedure is the critical difference between the distribution of the t-statistic as to whether  $\beta < 1$  or  $\beta = 1$ . To overcome this problem Van Dijk and Schotman (1991) propose to use a Bayesian approach which avoids the discontinuity. The discontinuity also disappears if we drop the crucial assumption that the process  $\{s(t)\}$  was initiated in the indefinite past. If the process has been initiated at some point  $t-k$ ,  $k$  finite, e.g. the time of the breakdown of the Bretton-Woods system say, then  $s(t)$  has a proper distribution under  $H_0$  and  $H_1$ . The question thus arises which assumption provides the better approximation to reality.

To further investigate this question extend eq. (13) by adding MA-terms. Recently this extension has received considerable attention, see e.g. Christiano and Eichenbaum (1990), because it points to a methodological problem with the unit root testing procedures. The presence of MA terms raises the possibility of common factors, which

makes that the class of unit root and stationary processes are not meaningfully distinct in finite samples. The argument is fully developed in Blough (1989, 1990) and Cochrane (1991a), but we proceed by the simple example of Stock (1990).

Consider the following ARIMA model

$$(2.14) \quad s(t) - \beta s(t-1) = \epsilon(t) - \alpha \epsilon(t-1),$$

where the  $\epsilon(t)$  are i.i.d. Setting  $\beta = \alpha = 0$  yields the stationary model

$$(2.15) \quad s(t) = \epsilon(t).$$

Changing  $\beta$  to  $0 < \beta < 1$ , gives the stationary model of eq. (13). But if we set  $\beta = 1$ , this produces the nonstationary random walk model of eq. (1). So far, so good. Now introduce  $\alpha$ ,  $0 < \alpha < 1$ , and we get the nonstationary model

$$(2.16) \quad \Delta s(t) = \epsilon(t) - \alpha \epsilon(t-1).$$

However, if we choose  $\beta = \alpha = 1$ , then eq. (16) becomes the stationary model

$$(2.17) \quad \Delta s(t) = \Delta \epsilon(t).$$

This is just the first differenced version of eq. (15) in which the unit roots of the AR and MA part cancel (the common factor).

Note that in eq. (13) there is a range of  $\beta$ 's for which  $s(t)$  will be close to the  $s(t)$  from eq. (1), assuming that both the processes have been initiated at some date finitely far back in the past. Conversely, there is a range of  $\alpha$ 's for which  $\Delta s(t)$  of eq. (16) will be close to  $\Delta s(t)$  generated by eq. (17). This can be used to show that in finite samples any unit root process can be arbitrarily well approximated by stationary processes, and any stationary process can be arbitrarily

well approximated by unit root processes. This result carries over to (continuous) statistics which are based on the data that are generated by these processes. Thus, according to Blough (1990): "There are stationary processes under which statistics have distributions approximating those under a random walk, and there are unit root processes under which statistics have distributions approximating those under white noise". See also Campbell and Perron (1991) and the ensuing discussion. This implies that in finite samples any test of the null hypothesis of a unit root with size  $\phi$  can have power no greater than  $\phi$  against any stationary alternative and vice versa<sup>9</sup>. The implication of this result is not that persistence cannot be detected, or is unimportant. Only that it is not useful with finitely many observations to distinguish between the perfect arbitrage equation (1) or the case of almost perfect arbitrage with  $\beta$  slightly less than 1.

Empirically conventional or Dickey-Fuller critical values usually do not reject the null of a unit root. Hence there is ample evidence of a high persistence in  $s(t)$ . The only thing we cannot say is whether exactly  $\beta = 1$  or that  $\beta$  is slightly less than 1. A value of  $\beta$  slightly below 1 not necessarily invalidates the efficient market hypothesis either, because the opportunities for arbitrage may still be too small to be profitable (given transactions costs, available funds, etc.). Values above 1 lead to explosive processes, but the data do not show any indication for this. (Even if  $\beta$  were just slightly larger than 1, this would rapidly show up in the data, see fn. 10 below).

There have been other univariate tests of market efficiency, especially geared towards the risk premium. Among these are the popular variance bounds tests and the Euler equation tests. Both boil down to

---

<sup>9</sup> Some caution should be exercised because the two cases  $\alpha$  approaching 1 and  $\beta$  approaching 1 are not entirely symmetric. Sargan and Bhargava (1983) show that if  $\alpha = 1$ ,  $\hat{\alpha} = 1$  with asymptotic probability .65, and this result holds also in small samples with only somewhat smaller probabilities.

tests of particular discount rate models, and their evidence is equivalent to return forecasting regressions like the ones above, see Cochrane (1991b). Both models are rejected by the data, but this does not imply market inefficiency, only a rejection of the particular model that is used to test for market efficiency. Thus more elaborate modelling of e.g. risk aversion may yield models which are not rejected by the data.

Apart from univariate tests of efficiency, one can extend the information set to the history of related prices like spot prices on other currencies and test for efficiency in a multivariate setting. If individual spot rates contain a unit root, i.e. adhere to eqs. (1) and (2) such that efficiency cannot be rejected, a linear combination of the different rates, though, may be stationary. In this case the variables are said to be cointegrated. By way of example, recall the triangular arbitrage condition (fact 2) which states that the difference between two different dollar spot rates approximately yields the cross rate:  $s_1(t) - s_2(t) \approx s_3(t)$ . Thus there exists a linear dependency between these three rates and hence they are cointegrated. How is this compatible with efficiency of the foreign exchange market? Note that for three currencies there are only two relative prices, i.e. there can be only two independent markets. Hence the cointegrating relation induced by triangular arbitrage does not contradict efficiency. Thus in a multivariate setting to be consistent with efficiency, cointegration should be present if the prices relate to the same assets, while there should be no cointegration if the prices relate to different assets, see Granger (1986) who originally developed this idea.

Because the concept of cointegration is so important for the study of market efficiency, we will further investigate the issue for the relation between spot and forward rates. Recall the unit root process

(1), with  $\epsilon(t)$  stationary. Thus  $s(t)$  is nonstationary<sup>10</sup>, while the linear combination  $s(t+1) - s(t)$  is stationary. Now replace  $s(t)$  with the forward rate which is known to be nonstationary as well. Then  $s(t)$  and  $f(t)$  are said to cointegrate if  $s(t) - \beta f(t)$  for some value of  $\beta$  is stationary. The idea is that while individual variables may wander off, arbitrage keeps the two variables close to each other, i.e. both variables remain close to the long run equilibrium relation  $s = \beta f$ .

To formalize these ideas, we start again and consider the system

$$(2.18) \quad \begin{aligned} s(t+1) - s(t) &= f(t) - s(t) + \theta(t+1), \\ f(t+1) &= s(t+1) + \mu(t+1), \end{aligned}$$

where  $\theta(t+1)$  and  $\mu(t+1)$  are i.i.d. random variables. The second equation of the system is the covered interest parity condition (1.5), i.e.  $\mu(t)$  is the interest differential.<sup>11</sup> Taking expectations at time  $t$  of the first equation variables and using the second equation yields the uncovered interest rate parity equation (1.10). Both  $s(t)$  and  $f(t)$  are clearly nonstationary. Rewrite (18) into first differences using the difference operator  $\Delta x(t) = x(t) - x(t-1)$  as far as possible

$$(2.19) \quad \begin{aligned} \Delta s(t+1) &= [f(t) - s(t)] + \theta(t+1) \\ \Delta f(t+1) &= \theta(t+1) + \mu(t+1). \end{aligned}$$

Note that in the first equation we are left with an expression in levels  $f(t) - s(t)$ , which is the cointegrating long run equilibrium relation between  $s(t)$  and  $f(t)$ . The stationarity of  $f(t) - s(t)$  directly follows from the bottom equation in (18) by moving  $s(t+1)$  to

---

<sup>10</sup> As  $s(t+1) = \sum_{i=0}^{\infty} \epsilon(t+1-i)$ , the innovations in the distant past contribute as much to the position of  $s(t+1)$  as the more recent innovations. In explosive processes, e.g.  $s(t+1) = \lambda s(t) + \epsilon(t+1)$ ,  $\lambda > 1$ , the past is more important than the present.

<sup>11</sup> Instead of the i.i.d. assumption,  $\mu(t)$  may follow a stationary autoregressive process.

the left hand side, i.e.  $f(t+1) - s(t+1) = \mu(t+1)$  which is stationary by assumption. Also note that all other random variables in the system (19) are stationary as well. The presence of  $f(t) - s(t)$  in (19) makes that current deviations from the equilibrium relation  $s = f$  are corrected for by future opposite movements. For example, if  $s(t) > f(t)$ , then subtracting the second eq. from the first eq. in (19) and taking expectations gives  $E[\Delta s(t+1) - \Delta f(t+1)] < 0$ . For this reason (19) is often referred to as the error correction mechanism. In fact, if some variables are cointegrated, there always exists an error correction representation, see Engle and Granger (1987). Also note that this implies some kind of predictability. Even though  $\Delta f(t+1)$  is purely random, knowledge of the current levels  $s(t)$  and  $f(t)$  does help to predict  $\Delta s(t+1)$ , because  $E[\Delta s(t+1) | s(t), f(t)] = f(t) - s(t)$ . But, recalling fact 5, the predictability will not be very high.

The first equation of (19) has often been tested in regression analysis. However, almost always one has to reject the null hypothesis of a unitary coefficient for the error correction term. Most US dollar exchange rates even yield significantly negative coefficients. An extensive literature has developed, see e.g. Fama (1984), Baillie and McMahon (1989) and Froot and Thaler (1990). The book by Hodrick (1987) is entirely devoted to this topic and discusses several reasons for the negativity of  $\hat{b}$ . But as of to date the puzzle has not been resolved.

The error correction system contains a warning against a popular device in applied work. If the variables in a vector auto regression (VAR) are found to be nonstationary, then it is common practice to estimate the VAR in first differences. Note, however, this produces inconsistent estimates if some of the variables are cointegrated. Because omitting the term  $f(t) - s(t)$  in (19) produces the omitted variables problem. To illustrate how severe this problem can be, suppose one is interested in estimating the cointegrating coefficient  $\beta$  by regressing  $s$  on  $f$ . Assume  $(\theta, \mu)$  are i.i.d. uncorrelated

bivariate standard normal. Then a regression in levels yields (using

$$\frac{\sum f^2}{n} \xrightarrow{P} \frac{n(n-1)/2}{n}$$

$$\hat{\beta} = \frac{\sum s f}{\sum f^2} = \frac{\sum (f+\theta) f}{\sum f^2} \xrightarrow{P} \frac{\frac{n-1}{2} + 1}{\frac{n-1}{2}} \rightarrow 1.$$

and where  $p$  denotes convergence in probability.

The regression in first differences produces

$$\hat{\beta} = \frac{\sum \Delta s \Delta f}{\sum \Delta f^2} = \frac{\sum (\mu(-1)+\theta)(\mu+\theta)}{\sum (\mu+\theta)^2} \xrightarrow{P} \frac{\sum \theta^2/n}{\sum \theta^2/n + \sum \mu^2/n} \xrightarrow{P} \frac{1}{2}.$$

which is clearly downward biased. But note that if  $\text{var } \mu < \text{var } \theta$ , the bias will be smaller. This observation may be important because of fact 5. An example of the case where omitting the error correction term proved to be important is the issue of German dominance. Several researchers have estimated VAR's in first differences of European interest rates, but found no influence of German interest rates on the other rates. This effect was hypothesized to exist by several analysts of the EMS. However, Kirchgässner and Wolters (1992) recently showed that if error correction terms are included in the regressions the German dominance hypothesis cannot be rejected. Omitting these terms seriously biased the previous estimates.

Stock and Watson (1988), and Sims, Stock and Watson (1990) provide further details. Johansen (1991) develops a convenient testing procedure for the number of cointegrating relations. An accessible account of estimation procedures in case of cointegration is given by Lütkepohl (1991). Campbell and Perron (1991) provide further intuition. A final word of caution concerning testing for cointegration is in place. Because, just like in the univariate case, in finite samples the multivariate setting does not allow to distinguish between

stationary and unit root relations, the absence or presence of cointegrating relations only carries qualitative information about the presence or absence of persistence.

## 2.2. The Unconditional Distribution Function

There are at least three reasons for investigating the properties of the unconditional distribution of the returns on speculative investments. First, the shape of the unconditional distribution places restrictions on the form of the conditional distribution of  $\epsilon(t)$  given  $\epsilon(t-1), \dots, \epsilon(t-n)$ . These properties can often be ascertained in a more robust manner, due to e.g. the central limit law, than the peculiarities of the conditional distribution. Second, an important job of the financial analyst is to provide appropriate risk assessments of longer term risky projects. For this purpose statistics based on the unconditional distribution are useful. Third, returns are often employed in statistical procedures like regression analysis. The unconditional distribution gives a clue about the appropriate minimization criterion and uses of certain test statistics. The importance of these arguments derives from the characteristic fatness of the tails of the empirical distribution of returns, recall fact 7. The tails may be so thick that the second moment is not defined, thus impairing the appropriateness of e.g. the OLS regression procedure.

The fat tail property serves as an important organizing principle. At first the French mathematician Bachelier (1900) ventured that speculative prices follow a Brownian motion (i.e. a random walk in continuous time with normal distributed innovations). But the normality assumption clearly conflicts with the stylized facts. Therefore Mandelbrot (1963a,b) proposed to use the other members of the class of stable distributions, which, besides being fat tailed, also have the desirable invariance under addition property, recall facts 2 and 9, but fail to have a finite second moment. Less fat tailed distributions, like

the Student-t, see Blattberg and Gonedes (1974), and Engle's (1982) ARCH distributed innovations, see below, have been proposed because these models exhibit a finite variance. Other models exhibiting a higher than normal kurtosis, such as the discrete mixtures of normals studied in Kon (1984), the mixed diffusion jump process advanced by Press (1967) and the power exponential or GED discussed in Baillie and McMahon (1989), Hsieh (1989) and Nelson (1991) have been applied as well. Boothe and Glassman (1987) provides a comprehensive survey.

While these models in one way or another capture the higher than normal kurtosis, there is considerable controversy over the precise amount of probability mass in the tails of the distribution, e.g. whether or not the second moment is finite. Thus one would like to select the best model among these alternatives. Unfortunately, a comparison between the competing hypotheses is hampered by the fact that some of the models are non-nested (due to e.g. an infinite variance). Therefore conventional model selection criteria like the likelihood ratio or Cox tests cannot be used, see e.g. Loretan and Phillips (1992). Moreover, the concept of fat tails used in this literature is not made precise. To overcome these problems recent advances in the area of extreme value analysis can be usefully exploited, as this analysis focuses explicitly on the tail behavior of the distribution. To see how, consider a stationary sequence  $X_1, X_2, \dots, X_n$  of i.i.d. random variables with a common distribution function  $F$ . Suppose one is interested in the probability that the maximum

$$(2.20) \quad M_n = \max(X_1, X_2, \dots, X_n)$$

of the first  $n$  random variables is below a certain level  $x$ . This probability is given by

$$(2.21) \quad P\{M_n \leq x\} = F^n(x).$$

Extreme value theory studies the limiting distribution of the order statistic  $M_n$  scaled by two normalizing constants  $a_n$  and  $b_n$  :

$$(2.22) \quad P\{M_n - b_n \leq a_n x\} \xrightarrow{d} G(x),$$

where  $G(x)$  is a so called extreme value distribution and  $\xrightarrow{d}$  stands for convergence in distribution. If  $1-F(x)$  is regularly varying at infinity, recall eq. (1.6), choosing  $b_n = 0$  and  $a_n = F^{-1}(1 - 1/n)$ , then:

$$(2.23) \quad G(x) = e^{-x^{-\alpha}}, \quad \alpha > 0,$$

where  $\alpha$  is the tail index. Results for the case when the  $X_i$  are dependent are given in Leadbetter et al. (1983, ch. 3), and e.g. De Haan et al. (1989) for the particular case of ARCH innovations. The tail index is a good indicator of the tail fatness as it is related to the number of moments that exist. In fact a necessary condition for the limit in eqs. (22) and (23) is that  $F(x) < 1$  for all  $x$  and

$$(2.24) \quad \int_1^{\infty} x^{\beta} dF(x)$$

is finite for all  $\beta < \alpha$  and infinite for  $\beta > \alpha$ . This condition provides the following intuition concerning the index of regular variation. Loosely speaking the largest integer  $n < \alpha$  corresponds to the number of (integer) moments that exist. If the  $\beta$ -th moment exists, then (24) must certainly be integrable, while if (24) is not integrable, then the  $\beta$ -th moment does not exist. What about distributions  $F(x)$ , like the normal, for which all moments do exist? The exponential decline of the tails of a distribution like the normal makes that (24) is always integrable, and hence the limit (23) does not apply (recall that  $\exp(x)$  can be expanded as  $\sum x^j/j!$ ). For these type of distributions the limit law takes a different form:

$$(2.25) \quad G(x) = \exp(-e^{-x}).$$

It is easily shown by checking the regular variation property (1.6) that the Student-t and the heavy tailed stable model are in the domain

of attraction of  $G(x)$  in eq. (23).<sup>12</sup> For these two particular models the  $\alpha$  values correspond to the degrees of freedom and the characteristic exponent respectively. The ARCH model takes more effort as one has to use appropriate mixing conditions, see De Haan et al. (1989). Somewhat surprisingly, given the excess kurtosis, none of the other models discussed above are in the domain of attraction of  $G(x)$  due to their exponential declining tails, but do belong to the domain of attraction of the thin tailed extremal limit law (25). Thus there is a sharp difference between the two types of distributions, one class is thin tailed and the other has fat tails. Typically, exchange rate returns belong to the latter class.

The advantage of the extreme value approach is that all fat tailed models are nested with respect to their tail index into one model. The idea is then to estimate this index directly and use the asymptotic confidence interval to discriminate between the competing hypotheses. The tail index, given a number of observations  $X_i$ , can be estimated by maximum likelihood, see Smith (1987), or by a moment estimator. We will present the latter procedure, because it does not require the assumption that the highest observations exactly follow the law in (23) and therefore is more efficient. We present the intuitive derivation developed by De Haan (1990). Assume that  $X_1, \dots, X_n$  is a sample of independent realizations from a distribution  $F(x)$  with a regular varying tail. Thus

$$(1.6) \quad \lim_{t \rightarrow \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}, \quad \alpha > 0.$$

Suppose the density  $f(x)$  exists. Through integration by parts we have the following equivalence

---

<sup>12</sup> We say that  $F(x)$  is in the domain of attraction of  $G(x)$  if the limit (22) applies.

$$\begin{aligned}
 (2.26) \quad \int_1^{\infty} \frac{1-F(tu)}{u} du &= \log(u) [1-F(tu)] \Big|_1^{\infty} + \int_1^{\infty} \log(u) f(tu) t du \\
 &= \int_1^{\infty} [\log(tu) - \log(t)] f(tu) t du \\
 &= \int_t^{\infty} [\log(x) - \log(t)] f(x) dx .
 \end{aligned}$$

Combine (1.6) and (26) and apply the Lebesgue Convergence Theorem (interchanging the limit of the integral with the integral of the limit)

$$(2.27) \quad \frac{\int_t^{\infty} [\log x - \log t] f(x) dx}{1 - F(t)} =$$

$$\int_1^{\infty} \frac{1 - F(tu)}{1 - F(t)} \frac{du}{u} \rightarrow \int_1^{\infty} u^{-\alpha} \frac{du}{u} = \frac{1}{\alpha} .$$

Let  $X_{(n)} \geq X_{(n-1)} \geq \dots \geq X_{(1)}$  denote the ascending order statistics from the sample  $X_1, \dots, X_n$ . The idea is now to replace the left hand expression in eq. (27) by its sample analogue in order to estimate the inverse tail index  $\gamma = 1/\alpha$ . Let  $F_n(\cdot)$  denote the empirical distribution function. Thus, for some  $m$  take  $t = X_{(n-m)}$  and hence

$$\hat{\gamma} = 1/\hat{\alpha} = \frac{\int_{X_{(n-m)}}^{\infty} [\log x - \log X_{(n-m)}] dF_n(x)}{1 - F_n(X_{(n-m)})}$$

$$= \frac{\sum_{i=0}^{m-1} \log X_{(n-i)} - m \log X_{(n-m)}}{1 - \frac{n-m}{n}} \frac{1}{n}$$

so that

$$(2.28) \quad \hat{\gamma} = \frac{1}{m} \sum_{i=0}^{m-1} \log X_{(n-i)} / X_{(n-m)}$$

is the proposed estimator. This estimator was first developed by Hill (1975). Mason (1982) shows that  $\hat{\gamma}$  is a consistent estimator if  $m \rightarrow \infty$  and  $m/n \rightarrow 0$ . Hall (1982) and Goldie and Smith (1987) have shown that if  $m$  increases suitably rapidly and if the  $X_i$  are i.i.d., then asymptotically

$$(2.29) \quad (\hat{\gamma} - \gamma) \sqrt{m} \sim N(0, \gamma^2).$$

For an application of this methodology to foreign exchange rates see Koedijk, et al. (1990), and Hols and De Vries (1991). Akgiray et al. (1988) employ the maximum likelihood method to estimate  $\gamma$  for black market exchange rate returns. Other uses of extremal analysis can be made as well, e.g. the analysis of stock market crashes in Jansen and De Vries (1991). Loretan and Phillips (1992) employ the Hill estimator (28) to pretest for the existence of the fourth moment in several return series before applying a standard or nonstandard sample split prediction test. The typical values of the tail index found in these studies is regime dependent. For the exchange rates which are more or less freely floating, the tail index hovers around 3 to 4, while intra EMS rates and other rates which involve some kind of fixity settle around 2, so that the variance may just exist or not exist.

Extremal value analysis also proves a theoretical backing for fact 9. Let  $X_i$  be an i.i.d. sequence with common d.f.  $F(x)$ . If  $1 - F(x)$  varies regularly at infinity, i.e. satisfies (1.6) with tail index  $\alpha$ , then  $\max(X_1 + X_2, \dots, X_{n-1} + X_n)$ , or the maximum of any finite convolution, follows the limit law (23); see Feller (1971, ch. VIII.8). Even though one often finds that the first two unconditional moments of the returns do exist, and hence that the central limit law applies, it does not follow that the tails of the (rescaled) summands become normal. Mandelbrot (1963a, b) based his choice for the (non normal) sum stable distributions on the preservation of the shape of the empirical distributions under addition. This proved to be too strong a condition on the moments (infinite variance), but tail additivity seems to hold.

### 2.3. The Conditional Distribution Function

For forecasting purposes the conditional distribution of the dependent variable is of paramount interest, rather than the unconditional distribution. As Bollerslev and Engle (1986) put it : "the use of the conditional mean explains the success of an economic time series model in forecasting". In the field of speculative price movements most research focuses on the first two conditional moments. This follows from the fact that in any theoretical economic analysis of risk, the mean return and the variance, if defined, are the two parameters of central interest. The variance signifies the risk and the mean indicates the expected return on investment.

If the first two unconditional moments of the innovation in the returns  $\epsilon(t)$  in eq. (1.1) exist then the time series analysis may be greatly facilitated by the Wold decomposition.

**Theorem 2.2 (Wold).** Let  $\{\epsilon(t)\}$  be a covariance stationary process with  $E[\epsilon(t)] = 0$  and no deterministic components<sup>13</sup>. Then  $\epsilon(t)$  can be written as

$$(2.30) \quad \epsilon(t) = \sum_{j=0}^{\infty} \gamma_j x(t-j) .$$

where  $\gamma_0 = 1$ ,  $\sum_{j=0}^{\infty} \gamma_j^2 < \infty$ ,  $E[x(t)] = 0$ ,

$$E[x(t)^2] \geq 0, \quad E[x(t) x(t-j)] = 0 \text{ for } j \neq 0. \quad \square$$

For a lucid proof, see Sargent (1979, p. 257). The usefulness of this decomposition is that it states that any covariance stationary process can be expressed as an infinite MA process. This representation can often be well approximated through some finite ARMA process. Hence the popularity of ARMA and VARMA modelling. But one has to keep in mind that this only provides an approximation, the quality of which is restricted by possible nonlinearities in the data generating mechanism and the finiteness of the available dataset.

In the univariate context, and given the Wold decomposition theorem, not surprisingly, research at first focused on the autocorrelation pattern of the returns, see e.g. Fama (1965). Little or no autocorrelation was found even in the highest frequency data (minute to minute or day to day). This was interpreted as a confirmation of market efficiency. To match these empirical observations with theoretical results we ask the question why the Wold decomposition is of no avail for the study of efficient markets. Suppose  $\{s(t)\}$  is a martingale and hence satisfies the no arbitrage condition, or, alternatively, we say that  $\{\epsilon(t)\}$  is a fair game. From (30) we may write (recall  $\gamma_0 = 1$ )

---

<sup>13</sup> That is to say, it contains no component which can be predicted arbitrarily well from past realizations through linear least squares projections. If such a component is present, it has to be added to eq. (1.1).

$$\begin{aligned}
x(t) &= \sum_0^{\infty} \gamma_j x(t-j) - \sum_1^{\infty} \gamma_j x(t-j) \\
&= \epsilon(t) - E_{t-1}[\epsilon_t] \\
&= \epsilon(t),
\end{aligned}$$

as  $E_{t-k}[\epsilon(t)] = 0$  for any  $k \geq 1$  if  $\{s(t)\}$  is a martingale. Moreover, this shows

$$\begin{aligned}
\gamma_k x(t-k) &= \sum_k^{\infty} \gamma_j x(t-j) - \sum_{k+1}^{\infty} \gamma_j x(t-j) \\
&= E_{t-k}[\epsilon_t] - E_{t-k-1}[\epsilon_t] \\
&= 0
\end{aligned}$$

for any  $k \geq 1$ . But the decomposition is rather trivial as  $\epsilon(t) = x(t)$ . Hence the Wold decomposition is of no use for the study of martingales.<sup>14</sup> Therefore  $\gamma_0 = 1$ ,  $\gamma_k = 0$  for  $k \geq 1$  is the unique Wold decomposition of the ARCH process for example.

Now recall fact 10 on volatility clusters. This indicates that although univariate speculative price series are typically not autocorrelated in the mean, they are nevertheless characterised by dependency in the second moment. Mandelbrot (1963) already noticed that there are clusters of high and low volatility in the return data. Since then many authors registered the presence of a time varying volatility in different kinds of financial data. Not until the ARCH (Auto Regressive Conditional Heteroskedastic) model introduced by Engle (1982) and the GARCH (Generalised ARCH) extension developed by Bollerslev (1986) have economists come to grips with this phenomenon.

---

<sup>14</sup> I am grateful to Luc Lauwers for suggesting this presentation of the proof.

Traditionally, heteroskedasticity was approached by introducing an exogenous variable to explain the changing variance. The innovation introduced by Engle was not to try to explain the changing variance by an exogenous source, but to describe it on basis of the own history of the series (which is in the same spirit as the ARMA methodology for modeling the mean).

The ARCH(1) model for the exchange rate specification (1) reads

$$(2.31) \quad s(t) = s(t-1) + \epsilon(t),$$

$$\epsilon(t) = X(t) H(t)^{1/2},$$

$$H(t) = \omega + \lambda \epsilon(t-1)^2, \quad 0 < \lambda < 1,$$

$$X(t) \text{ i.i.d. } N(0,1).$$

Here the conditional distribution of  $s(t)$ , given  $\epsilon(t-1)$  and  $s(t-1)$  is normal, with mean  $s(t-1)$  and variance  $H(t)$ . The conditional variance  $H(t)$  is a function of the lagged squared innovations. This induces the clusters of high and low variance. To see this, square the innovation function in eq. (31) and substitute the variance function in this equation

$$(2.32) \quad \epsilon(t)^2 = \omega X(t)^2 + \lambda X(t)^2 \epsilon(t-1)^2.$$

Hence  $E[\epsilon(t)^2 | \epsilon(t-1)^2] = \omega + \lambda \epsilon(t-1)^2$ , and similarly for the conditional variance because  $E[\epsilon(t) | \epsilon(t-1)] = E[X(t)] E[(\omega + \lambda \epsilon(t-1)^2)^{1/2}] = 0$  by the independence of the  $X(t)$ . This latter argument shows that  $\{s(t)\}$  is still a martingale, but not a random walk because the  $\epsilon(t)$  are not i.i.d. As we saw above the martingale property is the crucial no arbitrage condition for market efficiency, but also renders the Wold decomposition rather useless. Here the ARMA methodology is transferred to modelling the second moment, instead of the first moment.

The ARCH scheme also induces the fat tail property on the unconditional distribution of the returns. To see this, recall eq. (32) which is in fact a first order stochastic difference equation. From a result in Kesten (1973) it is known that this equation has a solution

$$(2.33) \quad \epsilon(t)^2 \stackrel{d}{\rightarrow} \sum_{j=0}^{\infty} \omega X(t-j)^2 \prod_{i=0}^{j-1} \lambda X(t-i)^2$$

which is unique in distribution provided that

$$(2.34) \quad E[(\lambda X^2)^{\alpha/2}] = 1$$

for some  $\alpha > 0$ . Given the normality of  $X$ , we can solve for  $\alpha$  if  $0 < \lambda < 1$  by using

$$(2.35) \quad \Gamma\left(\frac{\alpha}{2} + \frac{1}{2}\right) = \sqrt{\pi}(2\lambda)^{-\alpha/2}.$$

As it turns out the law of  $\epsilon(t)$  in (33) is in the domain of attraction of  $G(x)$  in eq. (23), see De Haan et al. (1989), with tail index  $\alpha$  as computed in (35). Thus exactly  $\alpha$  moments are finite<sup>15</sup>. Hence the unconditional distribution of  $s(t) - s(t-1)$  is fat tailed.

Specifications like (31) have become extremely popular in the area of international finance. The reason is that it conveniently captures both the clustering phenomenon and the fat tail property. It can be used to explain e.g. the existence of a time varying risk premium, see Baillie and Bollerslev (1990), but also see Frankel and Meese (1987) who note that the variation in the variance is generally still too small to explain the variability of the risk premium. Some straightforward extensions of the scheme (31) are the use of nonnormal innovations, but fat tailed distributed  $X(t)$ . So that both the conditional

---

<sup>15</sup> The relation between  $\alpha$ ,  $\lambda$ , the variance and fourth moment are as follows. If  $\lambda < 1$ , then  $\alpha > 2$  and the variance is finite. If  $\lambda < \sqrt{1/3}$ , then  $\alpha > 4$  and the fourth moment is finite.

and unconditional distribution of the returns become fat tailed<sup>16</sup>. There is evidence that this yields a better description of the returns. Bollerslev (1986) suggested to add lagged  $H(t)$  terms to right hand side of the variance function, e.g.

$$(2.36) \quad H(t) = \omega + \lambda \epsilon(t-1)^2 + \beta H(t-1).$$

This model can be considered the variance analogue of the ARMA model and was dubbed GARCH (generalized ARCH)<sup>17</sup>. Of course more than one lag can be considered. In empirical studies one often encounters that  $\lambda + \beta$  is close to 1, cf. Table 1.5. Hence the fourth moment may not exist and one has to exercise care in reporting test statics that require a finite fourth moment. This is e.g. the case in testing procedures for serial correlation in the presence of ARCH, see Diebold (1987, p. 26). (Because one estimates parameters of the variance function, in testing one needs the 'variance of the variance' for the central limit law to be applicable). Nelson (1991) considers an extension whereby the logarithm of  $H(t)$  is a function of past  $X(t)$  and which alleviates some of the problems with the ARCH specification. See Hsieh (1989) for an application. De Vries (1991) also uses  $X(t)$  rather than  $\epsilon(t)$  in the variance equation, but assumes  $X(t)$  is non-normal stable distributed. This induces the desirable additivity property in the ARCH model. Drost and Nijman (1991) investigate the same issue by weakening the GARCH equation (36) to linear least squares projections of  $\epsilon(t)^2$  on  $\epsilon(t-1)^2$ ,  $\epsilon(t-2)^2$ , etc., and show this class of ARCH models is closed under addition.

The success of the ARCH model is that it cogently captures the volatility clusters (fact 10) and exhibits the fat tail phenomenon (fact 7),

---

<sup>16</sup> If e.g. the Student-t distribution with  $v$  degrees of freedom is used then  $\lambda < 1$  is still sufficient for a finite variance, but then  $\lambda^2 < (v-4)/(v-6) < 1/3$  is required for the fourth moment to be finite.

<sup>17</sup> The condition for a finite variance if  $X(t)$  is normal or Student distributed becomes  $\lambda + \beta < 1$ .

and is still compatible with the martingale (no arbitrage) structure of efficient market prices. Just like the ARMA methodology, the ARCH model does not use exogenous or fundamentals variables. This is its strength because of fact 16. At the same time it is the weakness of the ARCH model. Being an economist one would like to know how these clusters come about. As of to date we must admit we have little or no idea what causes the ARCH feature in the returns. There are some suggestions in the literature. For example Obstfeld (1987) has suggested the clusters arise from periodic changes in policy. To make this argument recall eqs. (10-12) and suppose that without intervention  $y(t) - y^*(t)$  is an i.i.d. random variable, while  $m(t) - m^*(t)$  follows a random walk :

$$(2.37) \quad m(t+1) - m^*(t+1) = m(t) - m^*(t) + \mu(t),$$

and  $\mu(t)$  i.i.d. It follows that

$$(2.38) \quad s(t) = m(t) - m^*(t) - \frac{\phi}{1+\psi} [y(t) - y^*(t)].$$

One policy rule is not to intervene in the money market. Another policy rule consists of income targeting :

$$(2.39) \quad m(t+1) - m^*(t+1) = m(t) - m^*(t) + [y(t) - y^*(t)] + \mu(t).$$

The solution for  $s(t)$  under this alternative policy rule remains as in (38). But  $\text{Var}[s(t+1)|s(t)]$  will be different under the two rules, due to the extra random variable on the right hand side of (39) vis-à-vis (37). Thus if there are clusters in the usage of certain policy variables, this could explain the ARCH feature of the returns. As of to date, however, there is little evidence for this, see e.g. Hodrick's (1989) perceptive study. Another suggestion is that the clustering may come from noise traders, see De Long et al. (1990). Frankel and Meese (1987) present some evidence based on survey data. But as of yet explaining ARCH is one of the more important open questions.

clusters  
 clusters

Another question is, whether the volatility  $\sigma$  generated by the ARCH process are compatible with market efficiency. Without some model of market equilibrium in the background this question is hard to address. We showed that the ARCH model (2.31) induces the martingale property on  $\{s(t)\}$  and hence there is no room for arbitrage. Now note that  $\text{Var}[s(t+1)|s(t)] = \omega + \lambda[s(t)-s(t-1)]^2$ . Define  $y(t) = [s(t)-s(t-1)]^2$ . It follows that  $E[y(t+1)|y(t)] = \omega + \lambda y(t)$ , which shows that  $y(t)$  cannot be a martingale. Thus if  $\text{Var}[s(t+1)|s(t)]$  is part of the utility function, the scheme  $\omega + \lambda y(t)$  may leave scope for arbitrage. The static (conditional) CAPM model with the ARCH effect of Bollerslev, Engle and Woolridge (1988) seems inadequate to deal with this issue. There do exist dynamic equilibrium asset pricing models with risk averse agents. See Le Roy (1973), Lucas (1978) and Abel (1988). Ohlsen (1977) has shown that risk aversion may imply that the mean returns are no longer a martingale, but not necessarily so. The implications for the (conditional) variance have not been scrutinized extensively. Thus an interesting open question is whether empirically ARCH constitutes a refutation of market efficiency, or is compatible with efficiency.

## REFERENCES

- ABEL, A.B., Stock prices under time-varying dividend risk: an exact solution in an infinite-horizon general equilibrium model, *NBER working paper* 2621, 1988.
- AKGIRAY, V., G.G. BOOTH and B. SEIFERT, Distribution properties of latin american black market exchange rates, *Journal of International Money and Finance*, 1988, 37-48.
- BACHELIER, L.J.B.A.V., *Théorie de la Spéculation* (Gauthier-Villars, Paris), 1900.
- BAILLIE, R.T. and P.C. McMAHON, *The Foreign Exchange Market : Theory and Econometric Evidence* (Cambridge University Press, Cambridge), 1989.
- BAILLIE, R.T. and T. BOLLERSLEV, "A multivariate generalized ARCH approach to modeling risk premia in forward foreign exchange rate markets, 4 *Journal of International Money and Finance*, 1990, 309-324.
- BARRO, R.J., "Inflationary finance and the welfare cost of inflation, *Journal of Political Economy*, 1972, 978-1001.
- BLATTBERG, R.C. and N.J. GONEDES, A comparison of the Stable and Student distributions as statistical models for stock prices, *Journal of Business*, 1974, 244-280.
- BLOUGH, S.R., The relationship between power and level for generic unit root tests in finite samples, *Working paper in economics* 232, The John Hopkins University, 1989.
- BLOUGH, S.R., Unit roots, stationary, and persistence in finite sample macroeconometrics, *Working paper in economics* 241, The John Hopkins University, 1990.
- BOLLERSLEV, T., Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 1986, 307-327.
- BOLLERSLEV, T. and R.F. ENGLE, Modeling the persistence of conditional variances, *Econometric Review*, 1986, 1-50.
- BOLLERSLEV, T., R.F. ENGLE and J.M. WOOLRIDGE, A capital asset pricing model with time-varying covariances, *Journal of Political Economy*, 1988, 116-131.
- BOOTHE, P. and D. GLASMANN, The statistical distribution of exchange rates : empirical evidence and economic implications, *Journal of International Economics*, 1987, 297-320.

- CAMPBELL, J.Y. and P. PERRON, Pitfalls and opportunities: What macro-economists should know about unit roots, *NBER Macroeconomics Annual*, 1991, 141-???
- CHRISTIANO, L.J. and M. EICHENBAUM, Unit roots in real GNP : Do we know, and do we care ?, *Carnegie-Rochester Conference Series on Public Policy*, 1990, 7-62.
- COCHRANE, J.H., A critique of the application of unit root tests, *Journal of Economic Dynamics and Control*, 1991a, 275-284.
- COCHRANE, J.H., Volatility tests and efficient markets : A review essay, *NBER working paper 3591*, 1991b.
- DE CEUSTER, M.J.K., The Relative Effectiveness of Efficiency Criteria, Ph.D. thesis, Universiteit van Antwerpen, 1992.
- DECUPERE, D. and DE GRAUWE, Psychological barriers in the foreign exchange markets, K.U.Leuven, *International Economics Research Paper 85*, 1991.
- DE GRAUWE, P., *International Money, Postward Trends and Theories*, (Oxford University Press, Oxford), 1989.
- DEKKERS, A.L.M. and L. de HAAN, On the estimation of the extreme-value index and large quantile estimation", *Annals of Statistics*, 1989, 1795-1832.
- DE LONG, J.B., A SHLEIFER, L.H. SUMMERS and R.J. WALDMANN, Noise trader risk in financial markets, *Journal of Political Economy*, 1990, 703-738.
- DIEBOLD, F.X., *Empirical Modeling of Exchange Rate Dynamics* (Springer-Verlag, Berlin), 1988.
- DROST, F.C. and T.E. NIJMAN, Temporal aggregation of GARCH processes, Tilburg University (mimeo), 1991.
- ENGLE, R.F., Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflations, *Econometrica*, 1982, 987-1007.
- ENGLE, R.F. and C.W. GRANGER, Co-integration and error correction, representation, estimation and testing, *Econometrica*, 1987, 251-276.
- FAMA, E.F., The behaviour of stock market prices, *Journal of Business*, 1965, 34-105.
- FAMA, E.F., Forward and spot exchange rates, *Journal of Monetary Economics*, 1984, 319-338.
- FELLER, W., *An Introduction to Probability Theory and its Application*, Vol. 1 (Wiley, New York), 1970.

- FELLER, W., *An Introduction to Probability Theory and its Applications*, Vol. 2 (Wiley, New York), 1971.
- FRANKEL, J.A., The diversifiability of exchange risk, *Journal of International Economics*, 1979, 379-393.
- FRANKEL, J.A. and R. MEESE, Are exchange rates excessively variable?, in S. FISCHER, ed., *NBER Macro Economics Annual*, 1987, 117-162.
- FROOT, K.A. and R.H. THALER, Anomalies: Foreign exchange, *The Journal of Economic Perspectives*, 1990, 179-192.
- FULLER, W., *Introduction to Statistical Time Series* (John Wiley & Sons, New York), 1976.
- GIELENS, G. and C.G. de VRIES, Speculative prices and stochastic processes, *Nieuw Archief voor Wiskunde*, 1990, 311-323.
- GOLDIE, C.M. and R.C. SMITH, "Slow variation with remainder : Theory and applications", *Quarterly Journal of Mathematics (Oxford 2nd Series)*, 1987, 45-71.
- GOLDSTEIN, M. and M. KAHN, Income and price effects in foreign trade, in R.W. JONES and P.B. KENEN, eds., *Handbook of International Economics*, vol. 2 (North-Holland, Amsterdam), 1985, 1041-1106.
- GOODHART, C.A.E. and L. FIGLIUOLI, Every minute counts in financial markets, *Journal of International Money and Finance*, 1991, 23-52.
- GOURIEROUX, C., J.J. LAFFONT and A. MONTFORT, Rational expectations indynamic linear models: Analysis of the solution, *Econometrica*, 1982, 409-426.
- GRANGER, C.W.J., Developments in the study of cointegrated economic variables, *Oxford Bulletin of Economics and Statistics*, 1986, 213-228.
- HAAN, L. de, S.I. RESNICK, H. ROOTZEN and C.G. de VRIES, Extremal behaviour of solutions to a stochastic difference equation with applications to ARCH-processes, *Stochastic Processes and their Applications*, 1989, 213-224.
- HAAN, L. de, Fighting the arch-enemy with mathematics, *Statistica Neerlandica*, 1990, 45-68.
- HAKKIO, C.S. and M. RUSH, Market efficiency and cointegration: An application to the sterling and Deutschemark exchange rates, *Journal of International Money and Finance*, 1989, 75-88.
- HALL, P., On some simple estimates of an exponent of regular variation, *Journal of the Royal Statistical Society, Series B*, 1982, 37-42.

- HANSEN, L.P. and R.J. HODRICK, Forward exchange rates as optimal predictors of future spot rates: an econometric analysis, *Journal of Political Economy*, 1980, 829-853.
- HAUSMAN, J., Specification tests in econometrics, *Econometrica*, 1978, 1251-1271.
- HILL, B.M., A simple general approach to inference about the tail of a distribution, *The Annals of Statistics*, 1975, 1163-1173.
- HODRICK, R.J., *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets* (Harwood Academic Publishers, Chur), 1987.
- HODRICK, R.J., Risk, uncertainty, and exchange rates, *Journal of Monetary Economics*, 1989, 433-459.
- HOLS, M.C.A.B. and C.G. de VRIES, The limiting distribution of extremal exchange rate returns, *Journal of Applied Econometrics*, 1991, 287-302.
- HSIEH, D.A., Modeling heteroskedasticity in daily foreign exchange rates, *Journal of Business and Economics Statistics*, 1989, 307-317.
- JANSEN, D. and C.G. de VRIES, On the frequency of large stock returns : Putting booms and busts into perspective, *Review of Economics and Statistics*, 1991, 18-24.
- JOHANSEN, S., "Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models", *Econometrica*, 1991, 1581-1590.
- KESTEN, H., Random difference equations and renewal theory for products of random matrices, *Acta Mathematica*, 1973, 207-248.
- KIRCHGASSNER, G. and J. WOLTERS, Does the DM dominate the Euromarket, (mimeo), 1992.
- KOEDIJK, K.G., M.M.A. SCHAFGANS and C.G. de VRIES, The tail index of exchange rate returns, *Journal of International Economics*, 1990, 93-108.
- KOEDIJK, K.G., P. STORK and C.G. de VRIES, Foreign exchange rate regime differences viewed from the tails, *Journal of International Money and Finance*, 1992, forthcoming.
- KON, S., Models of stock returns : A comparison, *The Journal of Finance*, 1984, 147-165.
- LEADBETTER, M.R., G. LINDGREN and H. ROOTZEN, *Extremes and Related Properties of Random Sequences and Processes* (Springer-Verlag, New York), 1983.

- LEROY, S.F., Risk-aversion and the martingale property of stock prices, *International Economic Review*, 1973, 436-446.
- LEROY, S.F., Efficient capital markets and martingales, *Journal of Economic Literature*, 1989, 1583-1621.
- LEVICH, R.M., Empirical Studies of Exchange Rates: Price Behavior, Rate Determination and Market Efficiency, in R.W. JONES and P.B. KENEN, eds., *Handbook of International Economics*, vol. 2 (North Holland, Amsterdam), 1985.
- LORETAN, M. and P.C.B. PHILLIPS, Testing the covariance stationarity of heavy-tailed time series: An overview of the theory with applications to several financial datasets, *SSRI paper 9208*, University of Wisconsin, 1992.
- LUCAS, R.E., Asset prices in an exchange economy, *Econometrica*, 1978, 1429-1445.
- LUTKEPOHL, H., *Introduction to Multiple Time Series Analysis* (Springer-Verlag, Berlin), 1991.
- MANDELBROT, B., New methods in statistical economics, *The Journal of Political Economy*, 1963a, 421-440.
- MANDELBROT, B., The variation of certain speculative prices, *The Journal of Business*, 1963b, 394-419.
- MASON, D.M., Laws of large numbers for sums of extreme values, *The Annals of Probability*, 1982, 754-64.
- MEESE, R. and K. ROGOFF, Empirical exchange rate models of the seventies: Do they fit out of the sample?, *Journal of International Economics*, 1983, 3-24.
- MUSSA, M., Empirical regularities in the behavior of exchange rates and theories of the foreign exchange market, *Carnegie-Rochester Conference Series on Public Policy*, 1979, 9-57.
- NELSON, D.B., ARCH models as diffusion approximations, *Journal of Econometrics*, 1990, 375-390.
- NELSON, D.B., Conditional heteroskedasticity in asset returns: A new approach, *Econometrica*, 1991, 347-370.
- OBSTFELD, M., "Peso problems, bubbles, and risk in the empirical assessment of exchange-rate behavior, *NBER Working Paper 2203*, 1987.
- OHLSON, J.A., Risk-aversion and the martingale property of stock prices: Comments, *International Economic Review*, 1977, 299-234.
- PRESS, S.J., A compound events model for security prices, *Journal of Business*, 1967, 317-335.

- ROSE, A.K., The role of exchange rates in a popular model of international trade: Does the "Marshall-Lerner" condition hold ?, *Journal of International Economics*, 1991, 301-316.
- SARGAN, J.D. and A. BHARGAWA, "Maximum likelihood estimation of regression models with first order moving average errors when the root lies on the unit circle, *Econometrica*, 1983, 799-820.
- SARGENT, T.J., *Macroeconomic Theory* (Academic Press, New York), 1979.
- SCHOTMAN, P. and H.K. van DIJK, A Bayesian analysis of the unit root in real exchange rate, *Journal of Econometrics*, 1991, 195-238.
- SIMS, C.A., J.H. STOCK and M.W. WATSON, Inference in linear time series models with some unit roots, *Econometrica*, 1990, 113-144
- SMITH, R.L., "Estimating tails of probability distributions, *The Annals of Statistics*", 1987, 1174-1207.
- STOCK, J.H. and M.W. WATSON, Variable trends in economic time series, *The Journal of Economic Perspectives*, 1988, 147-174.
- STOCK, J.H., Unit roots in real GNP: Do we know and do we care ? A comment, *Carnegie-Rochester Conference Series on Public Policy*, 1990, 63-82.
- TAYLOR, S., *Modelling Financial Time Series* (John Wiley, Chichester), 1986.
- VIAENE, J.M. and C.G. de VRIES, International trade and exchange rate volatility, *European Economic Review*, 1992, 1311-1321.
- VRIES, C.G. de, Theory and relevance of currency substitution with case studies for Canada and the Netherlands Antilles, *The Review of Economics and Statistics*, 1988, 512-515.
- VRIES, C.G. de, On the relation between GARCH and stable processes, *Journal of Econometrics*, 1991, 313-324.
- WASSERFALLEN, W., Flexible exchange rates a closer look, *Journal of Monetary Economics*, 1989, 511-521.
- WESTERFIELD, J.M., An examination of foreign exchange risk under fixed and floating rate regimes, *Journal of International Economics*, 181-200.

**PURDUE UNIVERSITY**

**CIBER**

***WORKING PAPERS***

**PURDUE CIBER - 1993 WORKING PAPERS**

---

---

- No. 93-101      Gordon M. Phillips, Robert J. Weiner  
*"Information and Normal Backwardation as Determinants of Trading Performance: Evidence from the North-Sea Oil Forward Market."* 1994. The Economic Journal.
- No. 93-102      Stephen R. Goldberg, Frank L. Heflin  
*"The Association Between the Level of International Diversification and Risk"*
- No. 93-103      John A. Carlson  
*"Risk Aversion, Foreign Exchange Speculation and Gambler's Ruin"*
- No. 93-104      John A. Carlson, Aasim M. Husain, Jeffrey A. Zimmerman  
*"Penalties and Exclusion in the Rescheduling and Forgiveness of International Loans"*
- No. 93-105      Kent D. Miller  
*"Industry and Country Effects on Manager's Perceptions of Environmental Uncertainties."* 1993. Journal of International Business Studies, 24: 693-714.
- No. 93-106      Stephen R. Goldberg and Joseph H. Godwin  
*"Foreign Currency Translation Under Two Cases-Integrated and Isolated Economies"*
- No. 93-107      Kent D. Miller  
*"A Comparison of Managers' Uncertainty Perceptions and Country Risk Indices"*
- No. 93-108      Jon D. Haveman  
*"The Effect of Trade Induced Displacement on Unemployment and Wages"*
- No. 93-109      Jon D. Haveman  
*"Some Welfare Effects of Dynamic Customs Union Formation"*
- No. 93-110      John A. Carlson, Insook Kim  
*"Central Banks' Expected Profits From Intervention"*

*If you would like to request copies of specific papers, please contact the Center for International Business Education and Research, Purdue University, Krannert School of Management, West Lafayette, IN 47907.  
(Phone: 317/494-4463 or FAX: 317/494-9658)*

**PURDUE CIBER - 1994 WORKING PAPERS**

---

- No. 94-001 Casper G. De Vries, Phillip A. Stork, Kees G. Koedijk  
*"Between Realignments and Intervention: The Belgian Franc in the European Monetary System"*
- No. 94-002 Casper G. de Vries, K. U. Leuven  
*"Stylized Facts of Nominal Exchange Rate Returns"*
- No. 94-003 Kent D. Miller  
*"Operational Flexibility Responses to Environmental Uncertainties"*
- No. 94-004 Kent D. Miller  
*"Economic Exposure and Integrated Risk Management"*
- No. 94-005 Kent D. Miller  
*"Diversification Responses to Environmental Uncertainties"*
- No. 94-006 John M. Hannon, Ing-Chung Huang, Bih-Shiaw Jaw  
*"International Human Resource Strategy and Its Determinants: The Case of Multinationals and Their Subsidiaries in Taiwan"*
- No. 94-007 John M. Hannon, Ing-Chung Huang, Bih-Shiaw Jaw  
*"International Human Resource Strategy and Control: The Case of Multinationals and Their Subsidiaries"*
- No. 94-008 John M. Hannon, Yoko Sano  
*"Customer-Driven Human Resource Policies and Practices in Japan"*
- No. 94-009 John A. Carlson, Insook Kim  
*"Leaning Against the Wind: Do Central Banks Necessarily Lose?"*
- No. 94-010 John A. Carlson, David W. Schodt  
*"Beyond the Lecture: Case Teaching and the Learning of Economic Theory"*
- No. 94-011 Alok R. Chaturvedi, Hemant K. Jain, Derek L. Nazareth  
*"Key Information Systems Management Issues in Developing Countries: Differences in the Indian and US Contexts"*

*If you would like to request copies of specific papers, please contact the Center for International Business Education and Research, Purdue University, Krannert School of Management, West Lafayette, IN 47907.  
(Phone: 317/494-4463 or FAX: 317/494-9658)*