THEORY AND RELEVANCE OF CURRENCY SUBSTITUTION WITH CASE STUDIES FOR CANADA AND THE NETHERLANDS ANTILLES

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Abstract—This paper develops the theory of currency substitution from a choice theoretic point of view. The main result offers a simple relationship between the relative amount of currencies held and their opportunity costs, i.e., interest and capital gains. Our hypothesis is tested by case studies for Canada and the Netherlands Antilles. In contradistinction with other studies, we conclude that the elasticity of currency substitution is very small and negative. It is shown how the omission of the interest term in other studies biases the elasticity of currency substitution considerably.

I. A Theory of Currency Substitution

Currency Substitution (further CS) can be treated from a macro- and a microeconomic point of view. Within the macroeconomic tradition CS has been modelled by means of money demand functions and portfolio balance models. Girton and Roper (1981) use the former methodology and Cuddington (1983) the latter. A less conventional route is taken by Miles (1978). Here we develop a micro based theory of CS.

In the choice theoretic literature one finds several approaches towards modelling the demand for money. Although the approaches differ considerably, all have in common that money plays a special role in the exchange process vis à vis the other commodities. Here we take it that money functions as a means to economize on the leisure time used in transactions. A transaction technology relates money to the time needed for exchanging goods and services. Saving (1971) offers a convincing account of this approach.

For a transaction technology to be plausible in a multicurrency world, we require it to exhibit the following features:

i. Any currency is in principle able to fulfill the medium of exchange role, as argued by Hayek (1978).
ii. Habit formation, "money is like language," and man-made constraints, limit and influence the usefulness of different currencies; see Clower (1967) and Tobin (1980).
iii. Diminishing marginal returns of cash balances, but the derived marginal utility from money should be finite for zero balances, to allow for barter. The transaction time increases with the number of real transactions.
iv. The implied money demand functions should exhibit the usual homogeneity properties.

We turn to the derivation of the opportunity costs of holding currencies based on the above transaction technology. Consider an individual who has a two period time horizon, faces an uncertain future exchange rate $e(j)$, and has to decide upon the consumption cum portfolio mix. The budget restraints read

$$\begin{aligned} &g - px - m - el - b - ed \geq 0, \\ &g(j) - p(j)x(j) + m + e(j)l + (1+r)b \\ &\quad + e(j)(1+i)d + e(j)k - fk - m(j) \\ &\quad - e(j)l(j) \geq 0, \quad j = 1, \ldots, n, \end{aligned} \tag{1}$$

where g is income, px denotes consumption expenditure, m and l are the domestic and foreign currencies, b and d are domestic and foreign bonds which pay interest r and i, respectively, and k refers to the amount of forward exchange purchased at time t for delivery at time $t+1$ against the forward rate f. The index j refers to the possible states of the world at time $t+1$.

Uncertainty at time t stems from the future spot rate, which takes on a value $e(j)$ with probability $\pi(j)$; $\Sigma\pi(j) = 1$. The time additive expected utility function V has consumption x and net leisure time $z - T$ as its arguments (ρ is the pure rate of time preference):

$$\begin{aligned} V = U&\left[x, z - T\left[S\left[\frac{m}{a}, \frac{el}{a}, h\right], x\right]\right] + \rho\Sigma_j\pi(j) \\ &U\left[x(j), z - T\left[S\left[\frac{m(j)}{a(j)}, \frac{e(j)l(j)}{a(j)}, h(j)\right], x(j)\right]\right] \end{aligned} \tag{2}$$

With $i - iv$ above in mind, we shape the transaction technology T as follows. Let T be a bounded convex function separable in S and commodities x, where S is a function of money and invoicing habits h. The function S is concave in currencies m and el and has positive first derivatives; however, T is declining in S. Money balances are deflated with a price index "a" to satisfy the homogeneity requirements.

Supposedly, the individual maximizes (2) subject to (1). Assume that the constraint qualification is met. Given the strict concavity of U and the concavity of S and minus T, this implies that the above problem has a

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solution. We state some of the necessary first order conditions:

$$U_t T_s S_{m/a} \frac{1}{a} - \lambda + \Sigma\pi(j)\lambda(j) \leq 0, \tag{3}$$

$$U_t T_s S_{el/a} \frac{e}{a} - \lambda e + \Sigma\pi(j)\lambda(j)e(j) \leq 0, \tag{4}$$

$$-\lambda + (1 + r)\Sigma\pi(j)\lambda(j) \leq 0, \tag{5}$$

$$-\lambda e + (1 + i)\Sigma\pi(j)\lambda(j)e(j) \leq 0, \tag{6}$$

$$\Sigma\pi(j)\lambda(j)e(j) - f\Sigma\pi(j)\lambda(j) \leq 0. \tag{7}$$

In (3)–(7) we have taken the partial derivatives of the Langrangian with multipliers λ with respect to m, l, b, d and k. These conditions are used to derive several propositions.

Equations (5)–(7) imply the *interest parity condition*: $f/e = (1 + r)/(l + i)$. With or without the presence of a forward market, (3)–(6) can be combined to find the marginal rate of substitution between currency holdings

$$\frac{S_{m/n}}{S_{el/n}} = \frac{1+i}{1+r} \cdot \frac{r}{i} = \frac{r}{i(f/e)}. \tag{8}$$

The interpretation of (8) is as follows. *With unrestricted capital markets, the opportunity loss in allocating a unit of income to the domestic currency instead of the domestic bond is the interest r foregone. The opportunity loss in allocating a unit of income to the foreign currency instead of the foreign bond is the interest i foregone times the capital gain f/e induced by expected exchange rate revaluations.* This result also indicates the effect of a change in opportunity loss on the currency mix. By the convexity properties of the transaction technology T, a rise in r, for example, would lower the currency ratio m/el. Thus, the result can be classified as a meaningful proposition which is open to falsification.

In the above, the opportunity losses have two components: interest and capital gains. With some exceptions, like Girton and Roper (1981), the interest component is disregarded or measured erroneously by quite a number of articles on CS. Examples are Miles (1978), Ortiz (1983), Spinelli (1983), Padoa-Schioppa and Papadia (1984), and Marquez (1987). Ortiz's argument for his reliance on the capital gains term, is that for Mexico the expected depreciation dominates the interest term. This may be true, however, for the case scrutinized by Miles, the substitution of US-$ for Canadian-$, we do expect that the interest term is important. According to Miles (1978, pp. 433–434), if money balances are borrowed in period t, then in order to repay the loan, the interest plus the principal must be held in period $t + 1$, ergo the relative price of two currencies is $(1 + r)/(1 + i)$. The slip in this argument is that the principal was received in the first period and should therefore be deducted from the costs as a benefit,[1] i.e., the flow of monetary services is maximized subject to a stock constraint.

Lastly, note that *the currency mix in (8) is independent from an individual's tastes and risk attitudes.*[2] Separability of U and T and the presence of the bonds markets allow the purchasing power risk of money to be arbitrated away. This is useful in the empirical evaluation of (8).

II. Specification, Estimation and Testing

The properties of the transaction technology T motivate the functional form for the empirical application. Recall that S is concave in m and l, given h. Suppose S conforms a CES-function to allow for different degrees of substitutability. This functional form has the required properties and allows one to formulate the competing models as nested hypotheses. Accordingly, S takes the following shape:

$$S = \left[V\left(\frac{m}{a}\right)^{\tau} + W\left(\frac{el}{a}\right)^{\tau}\right]^{1/\tau}, \tag{9}$$

where $-\infty \leq \tau \leq 1$ and $h = (V, W)$. The weights V and W reflect the institutional setting and invoicing habits h of the whole economy. Therefore, it is reasonable to relate V and W only to aggregate variables. Note that the currency ratio m/el will be relatively high if the ratio V/W is high.

Combine the marginal rate of substitution (8), and (9) into

$$m = \left(\frac{W}{V}\frac{r}{i}\frac{1+i}{1+r}\right)^{\sigma} el, \tag{10}$$

where $\sigma = 1/(\tau - 1)$ is the Elasticity of Currency Substitution (further ECS) and $-\infty \leq \sigma \leq 0$. Provided that W/V does not depend on individual specific variables, this relationship holds in the aggregate as well. Summing over individuals and taking logarithms gives

$$\ln\frac{M}{eL} = -\sigma \ln\frac{V}{W} + \sigma \ln\frac{r}{i}\frac{1+i}{1+r}, \tag{11}$$

where M and L are the aggregate stock of domestic and foreign currency held domestically.

To specify V and W, several arguments are considered. An important determinant of the demand for foreign currency balances is international trade. According to Grassman's law the greater part of foreign trade is invoiced in the seller's country currency. Therefore, the export–import ratio may explain changes in the currency ratio. Habits, like the invoicing practices, tend

[1] See Barnett (1978) for a discussion of the concept of the user cost of money in the literature on "near monies." However, Barnett's manipulation of budget constraints is not applicable here, due to exchange rate uncertainty.

[2] For a somewhat similar result, see Fama and Farber (1979).

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to change slowly. In order to model the habitual slackness in response to changes in the opportunity costs, lagged opportunity costs are used. The third argument concerns the returns to scale or language property of a currency. Supposedly, this feature can be captured by the aggregate money stocks held in the past.

The inclusion of lagged opportunity costs and currency ratio also has an empirical basis. Preliminary work investigated the presence of unit roots in any of the variables.[3] On the basis of the Dickey-Fuller test, the currency ratio and opportunity costs are non-stationary, but their first differences are stationary. The trade ratio is weakly stationary though, which seems plausible as it is commensurable with balance of payment changes and therefore with base money changes. Hence, the specification to be estimated employs first differences Δ of $\ln M/eL$ and $\ln r(1+i)/i(1+r)$, but logarithmic levels of the export–import ratio E/I:

$$\Delta \ln \frac{M}{eL} = \beta + \sigma\left(\Delta \ln \frac{r}{i}\frac{1+i}{1+r}\right) - \phi \ln\left(\frac{E}{I}\right)_{-1} + \epsilon,$$

which for ease of reference is written as

$$\Delta ml = \beta + \sigma(\Delta y + \Delta c) - \phi q_{-1} + \epsilon, \qquad (12)$$

where $ml = \ln M/eL$, $y = \ln r/i$, $c = \ln(1+i)/(1+r)$, $q = \ln E/I$ and ϵ is an error term.

As was discussed in the previous section, quite a number of articles disregard the interest component y. The specification then reads

$$\Delta ml = \beta + \sigma(\Delta c) - \phi q_{-1} + \mu. \qquad (13)$$

[3] Details on the empirical work are provided in De Vries (1986). Data are available from the author upon request. Data for Canada, from the Bank of Canada Review, were kindly provided by M. Miles. H. Lub was very generous in preparing the data for the Netherlands Antilles, from the Quarterly Bulletin of the Bank van de Nederlandse Antillen. U.S. data are from the U.S. Treasury Bulletin.

To compare (12) and (13), suppose $\beta = \phi = 0$, then the bias due to unjustified omission of the interest term Δy is $\sigma\hat{\gamma}$, where $\hat{\gamma}$ is the estimated coefficient of a regression of Δy on Δc.[4] Now, note that for small interest rates, the magnitude of σ is largely determined by the interest term y. To see this, differentiate (11) with respect to r and i:

$$d(M/eL)/(M/eL) = \sigma\, dr/(r + r^2) - \sigma\, di/(i + i^2). \qquad (14)$$

The right-hand side is well approximated by $\sigma\, d(r/i)/(r/i)$ for r and i small. Hence, the interest term is the dominating factor in explaining the relative changes in the currency ratio. What is the effect of omitting this term? Use the following approximations $c = i - r$ and $y = (r/i) - 1$ to find the magnitude of the bias. Suppose that the foreign interest rate is constant, then $\hat{\gamma} = -1/i$. Hence, we expect the bias $\sigma\hat{\gamma}$ to be *highly positive* for small i. This does not imply that (13) necessarily produces a bad fit or forecasts, as Δy and Δc are highly intercorrelated.

Equations (12) and (13) are estimated for Canada (further CA) and the Netherlands Antilles (further NA), see table 1. The CA data were first employed by Miles, covering the period which starts with the float of the CA-$ in 1970 and ends in 1975, and have become a test case for currency substitution models. Whereas the US-$ content of the CA money stock is about 6.5%, in the NA it averages about 17%. Moreover, the NA-guilder is pegged to the US-$ and no forward market exists. For these quite different monetary regimes it is of interest to find the ECS. The NA data run from 1976 to 1985 and have not been used before; due to a lack of trade data, q is omitted from the regressions for the NA.

[4] Conversely, the bias due to omitting the capital gains term is $\sigma\widehat{(1/\gamma)}$.

TABLE 1.—THE ELASTICITY OF CURRENCY SUBSTITUTION

$\Delta(y+c)$	Δc	q_{-1}	DW	ξ	LL
		Canada			
−.19[a]		.25[b]	1.61	16.16	65.18
(.10)		(.12)			
	2.41	.24[b]	1.62	15.75	64.68
	(1.62)	(.12)			
		Netherlands Antilles			
−.27			2.08	8.43	54.25
(.19)					
	2.55		2.07	8.25	54.18
	(1.82)				

Note: In the table the DW, ξ and LL are respectively the Durbin-Watson statistic, the post sample goodness-of-fit statistic and the logarithm of the likelihood function Standard errors are reported between parentheses. For CA 63 observations were used for estimation and 12 for prediction, for the NA these numbers are respectively 78 and 12. Two periods with outliers due to short-lived credit controls were omitted from the analysis for the NA. The explanatory variables for CA are lagged by one period

[a] Significant at the 10% level

[b] Significant at the 5% level

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From table 1 it appears that the ECS is negative and close to zero for both countries. *This indicates that the US-$ is a rather imperfect substitute for the CA-$ and the NA-guilder*. For CA this confirms casual experience. The low ECS for the NA may be due to the peg against the dollar ever since 1973. Concerning the two other coefficients in (13), the intercept never differed significantly from zero and the coefficient ϕ on invoicing practices proved to be significantly positive. Furthermore, the restriction $\phi = \sigma$ could not be rejected on the basis of the t-test. This size of $-\phi$ is lower than expected on the basis of Grassman's law, but plausible given that most of CA's trade is conducted in the US-$ (see McKinnon (1979, p. 74).[5]

The above conclusion is at variance with Miles' interpretation of the data. Using c as the sole regressor, Miles found a positive elasticity of 5.78. Repeating Miles' procedure by estimating (13), we find an elasticity of 2.41. One expects a positive elasticity in (13) vis à vis (12), as $\Delta(y + c)$ and Δc are negatively correlated Apart from the sign change, the absolute value of the elasticity is much higher and this leads to the conclusion that the US-$ and CA-$ are substitutes. However, the bias due to omitting the interest term Δy is $\sigma\hat{\gamma}$. For both countries $\hat{\gamma}$ is highly negative, respectively -15.82 for CA and -10.21 for the NA. This implies that the exclusion of Δy introduces a considerable upward bias, and $\hat{\gamma}$ accounts well for the different $\hat{\sigma}$ in table 1.

Given the size of the bias, one likes to find empirical support for Δy to be relatively important in explaining Δml vis à vis Δc, c.f. (14). However, both terms are highly intercorrelated for small values of r and i. This hampers estimation of the coefficients for an equation in which Δy and Δc enter separately. However, the relative importance of the two terms can be illustrated in the following way. First, one regresses Δy on Δc and computes the residuals $\Delta y - \hat{\gamma}\Delta c$. Second, these residuals and Δc are used in a regression to explain Δml. Note that these regressors are orthogonal, so that the coefficient on $\Delta y - \hat{\gamma}\Delta c$ measures how much Δy contributes to explaining Δml independently from Δc. The procedure is also reversed. Standardized beta-coefficients are reported in table 2 to indicate the relative importance of the two variables in explaining Δml. From table 2 it is clear that Δy as an independent source contributes more to the explanation of Δml, then does Δc as an independent source.

To conclude, theory predicts that the optimal currency mix depends on the opportunity costs of holding money and some habit formation variables. The opportunity costs consist of an interest term and a capital gains term. For low interest countries the interest term dominates. Estimates of the ECS were shown to have the wrong sign and size when based on a regression which includes the capital gains terms only.

[5] I owe this observation to a referee.

TABLE 2.—RELATIVE EXPLANATORY POWER

Variables	Standardized beta-coefficients	
	CA	NA
Δy	.78	.98
$\Delta c - \widehat{(1/\gamma)}\,\Delta y$	.22	.02
$\Delta y - \hat{\gamma}\Delta c$	.39	.22
Δc	.61	.78

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