ON THE FREQUENCY OF LARGE STOCK RETURNS: PUTTING BOOMS AND BUSTS INTO PERSPECTIVE

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Abstract—Numerous articles have investigated the distribution of share prices, and find that the returns are fat tailed. Nevertheless, there is still controversy about the amount of probability mass in the tails, and hence about the most appropriate distribution to use in modeling returns. This controversy has proven hard to resolve, as the alternatives are non-nested. We employ extreme value theory, focusing exclusively on the larger observations in order to assess the tail shape within a unified framework. We find that at least the first two moments exist. This enables one to generate robust probabilities on large returns, which put the recent stock market swings into historical perspective.

I. Introduction

THE October 1987 stock market plunge and its aftermath have led to numerous investigations. In contradistinction with the first reviews, currently most researchers regard the drop as a sizeable but not unlikely market correction for which no particular abnormal circumstances are to blame. Roll (1988), for example, finds no evidence that institutional factors were a major contributor, but describes the plunge as a normal response to an aggregate world market shock. The aim of this paper is to judge the events against the frequency with which extreme price changes are expected to occur. One way to approach this question is to employ one of the distributions that have been advanced in the literature for modeling stock price returns.[1] While these models in one way or another capture the higher than normal kurtosis, there is considerable controversy over the exact amount of probability mass in the tails of the distribution, e.g., whether or not the second moment is finite. This controversy is not immaterial to our problem, as the tail shape of the distribution is essential for determining the frequency of large returns. Unfortunately, a comparison between the competing hypotheses is hampered by the fact that some of the models are non-nested and exhibit infinite variance. Therefore, conventional model selection tests do not apply.[2]

In this paper we propose to address the issues in a novel way by explicitly focusing on the tails of the distribution through the limit laws for the distribution of maxima (minima). These limit laws nest the alternative probability models by the so-called tail index α. For example, the stable hypothesis has $\alpha < 2$, while the Student-t model allows for $\alpha \geq 2$. This statistic α can be estimated, its estimator is asymptotically normally distributed, and, hence, tests of hypotheses are possible. In analogy with the reliance on the central limit law for inferences about the mean, this approach has the advantage that it does not depend on a particular probability model as the maintained hypothesis.[3] A disadvantage is that α is not a sufficient statistic for the entire return distribution, but it is a sufficient statistic for the tail shape of the distribution of extreme returns. Given an estimate for α, one can establish the sizes of the extremely low or high returns that would be rarely exceeded.[4] This allows for extrapolation of the empirical distribution function, and thereby the events of October 1987 can be put into perspective.

II. Theory

Consider a stationary sequence $X_1, X_2, \ldots$ of independent and identically distributed (i.i.d.)

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[1] Among the better known models are Mandelbrot's (1963) stable hypothesis; the Student-t distribution considered by Praetz (1972), and Blattberg and Gonedes (1974); the discrete mixture of normals studied in Kon (1984); the mixed diffusion jump process advanced by Press (1967); the ARCH process introduced by Engle (1982); and the recent power exponential or GED proposed by Baillie and McMahon (1989).

[2] The likelihood ratio criterion employed by Blattberg and Gonedes (1974), and Kon (1984), for example, is uninformative as its distribution is unknown.

[3] Related work is by Akgiray and Booth (1988) who employ a maximum likelihood procedure. We use the more efficient nonparametric estimator.

[4] McCulloch (1981) calculates bankruptcy probabilities maintaining a specific distribution model.

random variables with a distribution function F (d.f. F). Suppose one is interested in the probability that the maximum

$$M_n = \max(X_1, X_2, \ldots, X_n) \tag{1}$$

of the first n variables is below a certain level x. As is well known, this probability is given by

$$P\{M_n \le x\} = F^n(x). \tag{2}$$

Extreme value theory studies the limiting distribution of the order statistic M_n (appropriately scaled). That is, one is interested under what conditions there exist suitable normalizing constants $a_n > 0, b_n$, such that

$$P\{a_n(M_n - b_n) \le x\} \xrightarrow{w} G(x), \tag{3}$$

i.e.,

$$F^n(x/a_n + b_n) \xrightarrow{w} G(x),$$

where $G(x)$ is one of the three asymptotic distributions that are defined below, and w stands for weak convergence. If (3) holds, we shall say that F belongs to the domain of attraction of G, and write $F \in D(G)$. The main result is the Extremal Types Theorem:

THEOREM 1: *The possible limiting d.f. G are*

$$\begin{aligned} \text{Type I: } G(x) &= \exp(-e^{-x}) & -\infty < x < \infty; \\ \text{Type II: } G(x) &= 0 & x \le 0, \\ &= \exp(-x^{-\alpha}), & x > 0; \\ \text{Type III: } G(x) &= \exp(-(-x)^{\alpha}) & x < 0, \\ &= 1 & x \ge 0; \end{aligned} \tag{4}$$

with the index $\alpha > 0$.

The index α is called the tail index, and for convenience we sometimes use its inverse $\gamma = 1/\alpha$. Mood et al. (1974, p. 261) provide an introductory account to this result.[5]

The advantage of using the limiting d.f. $G(x)$ is that no detailed knowledge of $F(x)$ is needed. A complication is the fact that there are three limit laws, and which one is applicable depends on the tail shape of $F(x)$. The qualitative characteristics of the economic process may, however, point to the relevant limit law. Consider the following two necessary conditions from De Haan (1976):

Condition 1: If $F \in D$ (Type I $G(x)$) and $F(x) < 1$ for all x, then $\int_1^\infty t^\beta \, dF(t)$ is finite for all β.

Condition 2: If $F \in D$ (Type II $G(x)$), then $F(x) < 1$ for all x and $\int_1^\infty t^\beta \, dF(t)$ is finite for $\beta < \alpha$ and infinite for $\beta > \alpha$.

The intuition behind these conditions is as follows. Loosely speaking, the tail of the distribution is either declining exponentially or by a power. In the first case all moments exist, but in the second case the higher moments do not decay rapidly enough when "weighed" by the tail probabilities to be integrable, i.e., the d.f. $F(x)$ has fat tails. This explains the appearance of the double exponential in the Type I limit law and the exponential format of the Type II law as well. The third limit law is characterized by the fact that it has a finite upper endpoint. Given that stock returns are strongly fat tailed, and unbounded in principle, the Type II limit law is the only relevant type.

A sufficient condition on $F(x)$ for the Type II limit to obtain is

Condition 3: It is sufficient for $F \in D$ (Type II $G(x)$) that it has no finite upper endpoint, and for each $x > 0$ and some $\alpha > 0$

$$\lim_{t\to\infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}.$$

The latter condition boils down to regular variation at infinity, see Feller (1971, ch. VIII.8). A very useful implication of regular variation is the following result from Feller (ch. VIII.8):

THEOREM 2: *If* $1 - F(x)$ *varies regularly at infinity, i.e., satisfies Condition* 3, *then the* M_n *from* $F(x)$ *or any finite convolution of* $F(x)$ *follow the same limit law.*

The following discussion shows how the above conditions and theorem may be employed in specific cases. Return to the d.f.'s $F(x)$ that have received most of the attention in the literature on stock returns. First, variates from the normal, discrete mixtures of the normal, the mixed diffusion jump processes (with finite jump parameter) and the power exponential distribution possess all

[5] Leadbetter et al. (1983) give a comprehensive treatment and also treat the case of dependency, when the scaling parameter a_n has to be modified by a constant multiplicative factor θ, $0 < \theta \le 1$. Most of the results below carry over to dependent variates, and therefore we do not treat this case explicitly.

moments, and in fact Condition 1 applies.[6] Given such a lack of probability mass in the tails, these alternatives seem unfit for modelling stock returns. Second, variates from the Student-t, the stable d.f. and the ARCH process satisfy Condition 2, i.e., not all moments are finite. To verify that the Student-t class satisfies Condition 3 is straightforward. See, for example, Mood et al. (1974, p. 262) for a proof. Note that the degrees of freedom v are equal to the tail index α in (4). The proof that the stable distribution satisfies Condition 3 takes a little bit more effort, but follows, e.g., from manipulation of the asymptotic expansions given in Feller (1971). In De Haan et al. (1989) it is shown, by using appropriate mixing conditions, that the ARCH process is also in the domain of attraction of the Type II limit law. Because the class of the stable distributions is closed under addition, Theorem 2 above holds trivially for the stable distributions. Fortunately, even though, e.g., Student-t distributions are not invariant under addition, their tail behavior is unaffected by aggregation. That is, M_n generated from a Student-t or any finite sum of Student-t variates all tend to follow the Type II limit law, with the same α! This provides a certain robustness to the empirical applications below, as one can rely on the highest frequency data available for estimating the tail index.

From this discussion it is immediate that the relevant $F(x)$'s are nested within the Type II limit law, and are distinguished by different values for α. Specifically, the leptokurtic stable hypothesis requires $\alpha < 2$ while the Student-t class and ARCH process have $\alpha \geq 2$. The idea is now to estimate α directly without a prior commitment to either hypothesis.

The estimation procedures for α fall into two categories. If the Type II limit law applies, direct estimation by maximum likelihood is consistent, but it is not the most efficient procedure, i.e., the Cramér-Rao bound does not apply because the limit law is not followed exactly in finite samples, see Smith (1987). Recently, more efficient estimators have been proposed based on the largest order statistics, which require only that the distribution generating these observations is well behaved. This implies that the remaining estimation error can be solely attributed to the use of finite samples. For example, regular variation at infinity is often a sufficient condition. The focus here is on these methods.

Let $X_1, X_2, \ldots,$ be a sequence of stationary i.i.d. observations from some distribution function $F \in D$ (Type II $G(x)$). We are interested in obtaining an estimate for γ, given that the Type II limit applies. Define $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ as the ascending order statistics from a sample $X_1, X_2, \ldots, X_n$ of n consecutive stock returns X_i. The proposed estimator reads

$$\hat{\gamma} = 1/\hat{\alpha} = \frac{1}{m} \sum_{i=1}^{m} \left[\log X_{(n+1-i)} - \log X_{(n-m)}\right]. \tag{5}$$

The statistic $\hat{\gamma}$ first appears in Hill (1975). Mason (1982) proves that if Condition 3 is satisfied, $\hat{\gamma}$ is a consistent estimator for γ. Consistency obtains as well for a nonindependent sequence of X_i's, if the dependency is not too strong. By a result in Goldie and Smith (1987), it follows that $(\hat{\gamma} - \gamma)m^{1/2}$ is asymptotically normal with mean zero and variance γ^2.

The estimation procedure requires $m(n) \to \infty$, but for a finite sample it is not known how to choose m optimally. In the empirical section we conduct a Monte Carlo study to select m. Due to the asymptotic normality of $\hat{\gamma}$, the MSE criterion may be used for selecting an optimal m for given sample size n and d.f. $F(x)$.

The implied asymptotic confidence interval allows one to test directly for competing hypotheses about $F(x)$, e.g., the stable and Student-t distributions. The former requires $0 < \alpha < 2$ and the latter allows for $\alpha \geq 2$. As noted, discrimination between the two hypotheses is hampered by their non-nestedness. However, as our estimate of α is not conditional upon one of the two hypotheses being true, the asymptotic confidence interval may be used to test for H_0: $\alpha < 2$ against H_1: $\alpha \geq 2$. The asymptotic normality of $1/\hat{\alpha}$ may also be exploited to compare α estimates from different samples. The following statistic Q:

$$Q = \left(\frac{\alpha_1}{\hat{\alpha}_1} - 1\right)^2 m_1 + \left(\frac{\alpha_2}{\hat{\alpha}_2} - 1\right)^2 m_2, \tag{6}$$

where the α and m are as in equation (5) and the subindexes refer to two independent samples, is

[6] By Theorem 1.6.1. in Leadbetter et al. (1983) it follows that all four processes belong to the domain of attraction of the Type I $G(x)$ and hence the inverse of the tail index γ is zero.

asymptotically $\chi^2(2)$ distributed. It can be used to test for stability over different subsamples.

Given an estimate for the tail index α, one can establish extreme return levels that are only rarely exceeded. This is achieved by extrapolating the empirical distribution function outside the sample domain. The following procedure only requires regular variation, and uses order statistics to estimate the excess levels $\hat{x}_p$ for which

$$P\{X_1 \leq \hat{x}_p, \ldots, X_k \leq \hat{x}_p\} = F^k(\hat{x}_p) = 1 - p,$$

for small p and given k. The following is a consistent estimator of the excess levels:

$$\hat{x}_p = \frac{(kr/pn)^{\hat{\gamma}} - 1}{1 - 2^{-\hat{\gamma}}}\left(X_{(n-r)} - X_{(n-2r)}\right) + X_{(n-r)}, \tag{7}$$

where n is the number of observations, k is the time period considered, $r = m/2$, and p is the probability of excess. The proof of consistency of $\hat{x}_p$ is given in Dekkers and De Haan (1989).

A heuristic interpretation of the estimator in equation (7) is as follows. The pattern of the empirical d.f. $F_n(x)$ as signified by the level $X_{(n-r)}$ and step size $(X_{(n-r)} - X_{(n-2r)})$ is extrapolated outside its domain by using the way the limit law extends. The latter is represented through the multiplication factor in front of the stepsize. For $p > 1/n$ the empirical d.f. is a good estimator for $\hat{x}_p$, due to its unbiased mean squared error consistency; see Mood et al. (1974, p. 507). But for $p < 1/n$, $F_n(x)$ is of no avail, and the above is a device to extend $F_n(x)$ beyond $1/n$.

III. Empirical Analysis

In this section we evaluate the leptokurtosis in stock returns and tabulate probabilities on observing excessively high and low returns. The data consist of 6000 daily dividend compensated stock returns for ten stocks from the S & P 100 list, and two market indices over the period 1962 to 1986.[7] There is little disagreement about the qualitative properties of stock returns. Typically, daily returns are a stationary series that is strongly leptokurtic and possibly exhibits some low order serial dependence. Thus, the data meet the criteria for application of the theorems and estimators of the previous section.

As noted, the problem with any tail index estimator like $\hat{\gamma}$ in equation (5) is that it is conditional upon the portion m/n of the sample used for calculating the statistic $\hat{\gamma}$. It is not known how to choose m in finite samples, so we conducted a Monte Carlo experiment to find the m-level, conditional upon a sample size n and d.f. $F(x)$, for which the MSE is minimal. We simulated with four different distributions: the Student-t distribution with 1, 2 and 3 degrees of freedom, and the inverted chi square distribution (see table 1). The first and last distribution are sum stable with characteristic exponents 1 and 1/2, respectively, and have more mass in the tails than the other distributions.

Clearly, the optimal m-levels vary inversely with the true tail index α. The reason is that the lower is α, the fatter the tails of the distribution, and hence the more "outliers" are available to estimate the tail index. Note that there is an asymmetry in table 1. For high m, increasing α deteriorates the MSEs more than lowering α for low m. This suggests that using too many observations such that some do not belong to the tail, but rather to the center of the distribution, is more

TABLE 1.—MSE, BIAS SQUARED AND m-LEVELS

m	$\alpha = 1/2$	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$
1680 ($\alpha = 1/2$)	0.0028 [0.21] (0.0024)	0.0491 [0.99]	0.1443 [1.00]	0.2079 [1.00]
470 ($\alpha = 1$)	0.0081 [0.00]	0.0020 [0.17] (0.0021)	0.0048 [0.88]	0.0138 [0.98]
170 ($\alpha = 2$)	0.0206 [0.00]	0.0054 [0.01]	0.0014 [0.29] (0.0015)	0.0031 [0.68]
100 ($\alpha = 3$)	0.0349 [0.01]	0.0092 [0.00]	0.0019 [0.015]	0.0023 [0.44] (0.0011)

Note: The left-hand column reports the m-levels conditional on the α value between the brackets for which the MSE is minimal. The MSEs for different (m, α) pairs are reported in the matrix, together with the bias squared as a percentage of the MSE between [.] brackets. The theoretical MSEs are reported between (.) brackets. The simulations consisted of 100 replications of 6000 draws. The α was estimated for $m = 10, \ldots, 2000$ with stepsize 10.

[7] Data were obtained from the CRSP Tape compiled by the Graduate School of Business of the University of Chicago. The stocks used are listed with the following ticker symbol: (1) IBM, (2) MOB, (3) MBK, (4) KM, (5) AMP, (6) HON, (7) NCR, (8) GW, (9) OI, and (10) BC. Furthermore two market indexes were used: the S & P 500 and the Unweighted Market Index, abbreviated to UMI. Returns were computed as log differences. Further details about the data and programs are available from the first author upon request.

harmful than not using all the available information. The bias part of the MSE, due to inclusion of the center characteristics, is thus seen to dominate the variance part, which stems from inefficient use of the available information.

On the basis of these results we decided to be conservative in choosing the number of observations for the actual estimation. The estimates for the tail index in table 2 are conditioned on $m = 100$. All estimates are between 3 and 5 and are significantly above 2 according to the 95% asymptotic confidence intervals.[8] We resolve the long-standing issue about the appropriateness of the Student-t or ARCH class vis à vis the stable class in favor of the former. The hypothesis that the returns follow a discrete mixture of normal distributions, a mixed diffusion jump process or the power exponential distribution is not tenable as the tail index is too small (significantly below 30, say). We conclude that stock returns are fat-tailed in comparison with the normal distribution, but still possess a finite variance. Thus the central limit theorem for addenda is applicable, but at the same time Theorem 2 above also applies.

One might argue that our estimates for the tail index are not robust, citing institutional changes on financial markets, c.f. Akgiray and Booth (1988, p. 52). In order to investigate this issue we split the sample into two parts, using April 26, 1973 as the dividing day. On this day the Chicago Board

[8] The estimates for the indices correspond to those of the individual stocks, as was anticipated on the basis of Theorem 2.

TABLE 2.—TAIL INDEX ESTIMATES

Stock	Upper Tail $\hat{\alpha}$	Maximum	Lower Tail $\hat{\alpha}$	Minimum
1	3.72 (2.99–4.45)	0.10	4.65 (3.74–5.54)	−0.09
2	3.64 (2.93–4.35)	0.11	3.68 (2.96–4.40)	−0.09
3	4.36 (3.51–5.21)	0.09	5.22 (4.20–6.24)	−0.10
4	3.76 (3.02–4.50)	0.12	3.60 (2.89–4.31)	−0.20
5	4.28 (3.44–5.12)	0.11	4.41 (3.54–5.28)	−0.15
6	3.34 (2.69–3.99)	0.13	3.53 (2.84–4.22)	−0.16
7	4.45 (3.58–5.32)	0.12	4.53 (3.64–5.42)	−0.13
8	4.62 (3.71–5.53)	0.21	4.11 (3.31–4.91)	−0.11
9	3.79 (3.05–4.53)	0.15	4.17 (3.35–4.82)	−0.10
10	4.56 (3.66–5.46)	0.26	3.71 (2.98–4.43)	−0.19
Index				
UMI	3.27 (2.63–3.91)	0.07	3.37 (2.71–4.03)	−0.05
S & P	3.96 (3.18–4.74)	0.05	4.30 (3.46–5.14)	−0.05

Note: Reported are the α estimates and their 95% confidence intervals for both the upper and lower tail; the maximum and minimum sample returns are given as well. The estimates were based on the estimator in (5) with $m = 100$, c.f. table 1.

TABLE 3.—STABILITY OF THE LOWER TAIL INDEX

Stock	1	2	3	4	5	6	7	8	9	10	UMI	S & P
Pre 73α	3.22	3.52	3.37	3.18	3.21	3.31	3.28	3.62	3.10	3.06	3.73	3.71
Post 73α	3.82	2.79	4.06	3.19	3.21	3.03	3.75	3.36	3.39	3.25	2.90	3.65
Upper	2.84	2.57	3.00	2.55	2.57	2.54	2.83	2.80	2.60	2.53	2.71	2.95
Lower	4.10	3.58	4.30	3.82	3.85	3.77	4.14	4.16	3.86	3.77	3.72	4.14

Note: The subsample α estimates are conditioned upon $m = 75$. This number is also used in computing the Q-test in formula (6), i.e., $m_1 = m_2 = 75$. The lower two rows give the lower and upper bounds of the interval for which $H_0 = \alpha_1 = \alpha_2$ is not rejected on the basis of the Q-test.

TABLE 4.—LOWER TAIL PROBABILITIES ON DAILY RETURNS

Returns	Probabilities					
	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5	Stock 6
−0.10	0.01952	0.14225	0.01092	0.16479	0.10972	0.25174
−0.20	0.00086	0.01299	0.00030	0.01469	0.00557	0.02475
−0.30	0.00013	0.00308	0.00003	0.00350	0.00095	0.00618
	Stock 7	Stock 8	Stock 9	Stock 10	UMI	S & P
−0.10	0.12805	0.10826	0.07519	0.51892	0.02165	0.00361
−0.20	0.00597	0.00553	0.00458	0.04555	0.00222	0.00019
−0.30	0.00097	0.00100	0.00087	0.01061	0.00057	0.00003

Note: The table is constructed by using formula (7), with $k = 260$, i.e., approximately covering one year, $r = 50$, $n = 6000$, and the α's are from table 2.

Option Exchange was organized. Moreover, it was the year of the final demise of the Bretton Woods agreement and the aggregate oil price shock. Table 3 reports the stability of the lower tail α's by means of the Q-test in equation (6). The stability of α is never rejected at the 5% significance level, as all the subsample estimates are within the bounds and thus the tails appear robust across different economic regimes. Similar results were obtained for the upper tail α's.

While knowledge of the value of the tail index is interesting in itself, the question of economic interest is how likely extreme returns are. To answer this question the α estimates from table 2 are used in formula (7). Table 4 gives negative return levels below which daily yields within the time span of one year would only move with the specified (low) probability. For example, the probability that *within a given year* stock 6 experiences a one day drop in its share price of more than 20% is 0.02475. Stated differently, on average, about once every $1/.025 = 40$ years, the share price of stock 6 will fall by more than 20%. From table 2 we know that the largest daily drop observed within the sample of 24 years was 16%. Hence, table 4 uses our knowledge about the tails of the distribution to extend our knowledge of stock returns over longer time spans and lower probabilities than empirically observed. By comparing the probabilities for different stocks, the table provides an alternative indicator for the amount of tail thickness. In this respect it can be used as a device for portfolio selection. Suppose an investor is interested in selecting the stock which minimizes the probability of extreme losses (i.e., the minimax strategy). Fixing the extreme losses at 30% or lower, the investor should select stock 3. Note that this stock also has the highest tail index estimate, i.e., $\hat{\alpha} = 5.22$.

The table also gives some perspective to the events of October 1987. On October the 19th many stocks and the market indices fell by more than 20%. From table 4 we see this is not an unlikely event for most of the individual stocks, and this is corroborated by the minima reported in table 2. What made this into a rare event was that all stocks dropped simultaneously by such a large amount, causing the sudden and severe drop in the market indices. According to table 4, however, this latter event carries a probability of no more than 0.0022, suggesting that crashes like the one of October 19 occur only once in about every 450 years. On the one hand, this probability vis à vis those recorded for the individual stocks evidences the effects of portfolio diversification whereby stock-specific shocks are mitigated. On the other hand, this probability is on the low side given that since 1899 there have been four days on which the Dow Jones industrial average (Dow) dropped by more than 10%. The relatively short sample we employed did not include these events and therefore may cause some underestimation.

Although the aggregate shocks covered by our sample period, like the oil shock of 1973, did not lead to excessive daily plunges, they did lead to sustained declines. To see whether extremal analysis does predict these sustained declines in the market indices, one could use returns over longer time spans than a day. Table 5 provides probabilities on observing *monthly yields* in excess of a return over the time span of *one year*. Note that the table is generated by using the daily $\hat{\alpha}$'s, which is permissible in view of Theorem 2. From table 5 we see that the probability that on aver-

TABLE 5.—PROBABILITIES ON MONTHLY RETURNS FOR THE UMI

Returns								
0.10	0.14	0.18	0.20	0.25	0.30	0.50	1.00	2.00
			Probabilities in Upper Tail					
0.7740	0.3493	0.1842	0.1395	0.0760	0.0456	0.0102	0.0012	0.0001
			Probabilities in Lower Tail					
0.5653	0.3209	0.1977	0.1589	0.0972	0.0633	0.0170	0.0023	0.0003

Note: The table was constructed by forming monthly yields from the daily yields through addition and then applying formula (7) with $k = 12$, i.e., covering a year, $r = 30$, $n = 294$ and the α's from table 2.

age within any year a monthly market drop exceeds 20% or 30% is 0.16 or 0.06, respectively, i.e., occurs once in about every 6 or 15 years, respectively. Hence, the probability of a sustained bearish market is quite high. It shows that while the events of October the 19th may be exceptional, a drop of similar proportions over a somewhat longer period is not unlikely.

IV. Conclusion

In this paper we investigate the tail behavior of stock returns, instead of looking at the entire distribution. Thereby one trades off knowledge about the center characteristics of the d.f. against the ability to nest d.f.'s with different probability mass in the tails by means of the limit laws for maxima. Empirical estimates of this encompassing model point towards the existence of a finite mean and variance but infinite higher moments, lending support to the Student-*t* and ARCH class distributions vis à vis the stable class and the mixtures of normal distributions.

The tail estimates were in turn used to generate probabilities and the associated extreme returns. Such tables are useful to investors who want to select a conservative portfolio. The tables also indicate the difference between investing in a specific stock or in a market portfolio. Not surprisingly, the risk on an extremely large or small yield is much higher for the former strategy than the latter.

REFERENCES

Akgiray, Vedat, and G. Geoffrey Booth, "The Stable-Law Model of Stock Returns," *Journal of Business & Economic Statistics* (Jan. 1988), 51–57.

Baillie, R. T., and P. C. McMahon, *The Foreign Exchange Market* (Cambridge: Cambridge University Press, 1989).

Blattberg, Robert C., and Nicholas J. Gonedes, "A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices," *Journal of Business* (Apr. 1974), 244–280.

Dekkers, Arnold L. M., and Laurens de Haan, "On the Estimation of the Extreme-value Index and Large Quantile Estimation," *Annals of Statistics* (Dec. 1989), 1795–1832.

Engle, Robert F., "Autoregressive Conditional Heteroskedastic Models with Estimates of the Variance of United Kingdom Inflation," *Econometrica* (July 1982), 987–1007.

Feller, William, *An Introduction to Probability Theory and Its Applications*, vol. 2 (New York: Wiley, 1971).

Goldie, Charles M., and Richard L. Smith, "Slow Variation with Remainder: Theory and Applications," *Quarterly Journal of Mathematics* (Oxford, 2nd series, 1987), 45–71.

Haan, Laurens de, "Sample Extremes: An Elementary Introduction," *Statistica Neerlandica* (1976), 161–172.

Haan, Laurens de, Sidney I. Resnick, Holger Rootzén, and Casper G. de Vries, "Extremal Behaviour of Solutions to a Stochastic Difference Equation with Applications to ARCH-Processes," *Stochastic Processes and their Applications* (Aug. 1989), 213–224.

Hill, Bruce M., "A Simple General Approach to Inference about the Tail of a Distribution," *The Annals of Statistics* (1975), 1163–1173.

Kon, Stanley, "Models of Stock Returns—A Comparison," *The Journal of Finance* (Mar. 1984), 147–165.

Leadbetter, M. R., Georg Lindgren, and Holger Rootzen, *Extremes and Related Properties of Random Sequences and Processes* (New York: Springer-Verlag, 1983).

Mandelbrot, Benoit, "New Methods in Statistical Economics," *The Journal of Political Economy* (Oct. 1963), 421–440.

Mason, David M., "Laws of Large Numbers for Sums of Extreme Values," *The Annals of Probability* (1982), 754–764.

McCulloch, J. Huston, "Interest Rate Risk and Capital Adequacy for Traditional Bank and Financial Intermediaries," in S. J. Maisel (ed.), *Risk and Capital Adequacy in Commercial Banks* (Chicago: University of Chicago Press, 1981).

Mood, Alexander M., Franklin A. Graybill, and Duane C. Boes, *Introduction to the Theory of Statistics* (New York: McGraw-Hill, 1974).

Praetz, Peter, "The Distribution of Share Price Changes," *Journal of Business* (Jan. 1972), 49–55.

Press, S. James, "A Compound Events Model for Security Prices," *The Journal of Business* (July 1967), 317–335.

Roll, Richard, "The International Crash of October 1987," *Financial Analysts Journal* (Sept. 1988), 19–35.

Smith, R. H., "Estimating Tails of Probability Distributions," *The Annals of Statistics* (1987), 1174–1207.